Introduction to Nonlinear Circuits - The Memristor

Dr. Bharathwaj "Bharath" Muthuswamy

Visiting Assistant Professor in Computer Science
The College of New Jersey
bharath.berkeley@gmail.com

October 25th 2017

About me...

- BS (2002), MS (2005), PhD (2009) in EECS from the University of California, Berkeley
  (advisors: Dr. Leon O. Chua, Dr. Pravin P. Varaiya)
  - For my MS, I worked on biomimetic bipedal robotics using Central Pattern Generators (I did not work on this after 2006)
  - For my PhD, my primary contribution was designing, implementing and rigorously proving the existence of chaos in the Muthuswamy-Chua system (circuit): an inductor-capacitor-memristor circuit in series (parallel)
- Areas of interest:
  - Nonlinear Dynamics (Circuits). Specifically: chaotic circuits and memristors
  - Embedded (FPGA) Systems and Education

Presentation Goal and Organization

- Goal: Discuss the memristor (4^{th} fundamental (non)linear circuit element)
- Organization: We will utilize ideas from my upcoming book (co-authored with Dr. Banerjee from Universiti Putra, Malaysia): Introduction to Nonlinear Circuits and Networks
  - The Discipline of Circuit Theory
  - Fundamental Circuit Variables
  - The Memristor
    - Mathematical Formulation (Gedanken-Experiment)
    - Properties
    - Physical Memristors: Ideal Memristor (Josephson junction) and Non-ideal memristors (discharge tubes, pn-junctions)
- Conclusion and Q/A

The Discipline of Circuit Theory

- What is circuit theory?
  - A branch of electrical engineering that is concerned with the terminal behavior of circuit elements
- Circuits vs. Networks

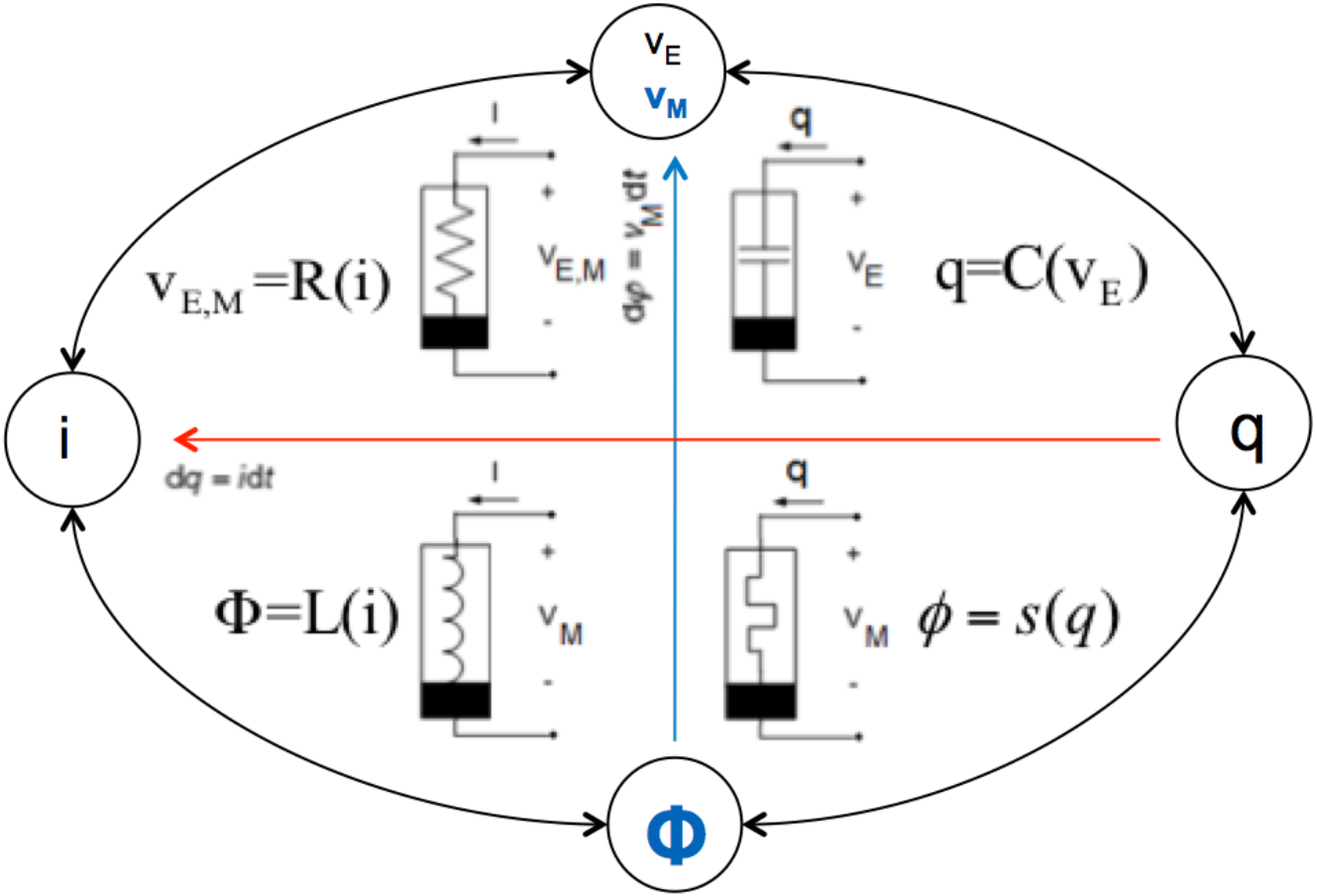
Linear vs. Nonlinear (SYSTEM)

- Note: Quantum Mechanics is fundamentally "linear"
  - So, how does nonlinear behavior (such as chaos) arise macroscopically?
- DEFINITION of a Linear SYSTEM
  - Principle of Superposition
- Question: Is the following system linear?

$$y = System(x) \stackrel{\triangle}{=} \alpha x + \beta \quad \forall \alpha, \beta \in \mathcal{R}; x, y \in [\mathcal{R} \rightarrow \mathcal{R}]$$

- Answer: NO!

Fundamental Circuit Variables



Memristor: Mathematical Formulation (Gedanken-Experiment)

- A memristor (menductor) defines a relationship between ϕ and q (q and ϕ):

$$\phi \triangleq s(q) \tag{1}$$

- In terms of $v - i$:

$$v(t) = M(q(t))i(t) \tag{2}$$

  Here, $M(q(t)) = M(\int_{-\infty}^{t} i(\tau)d\tau)$. M is the memristance function

- Note that a memristor is fundamentally nonlinear element (unlike the resistor, capacitor, inductor)
  - Do Eqs.(1) and (2) satisfy superposition?
  - When is the memristor linear?
    - A memristor is linear iff M is a constant $\Rightarrow$ Memristor is simply a linear resistor!

Memristor Properties

- We can generalize Eq.(2) (memristive system):

$$\dot{x} = f(x, i, t)$$
$$v = R(x, i, t)i \tag{3}$$

- Eq.(2) defines an ideal memristor, Eq.(3) defines a non-ideal memristor
- Probably the most relevant property for us: A memristor $v - i$ curve exhibits a pinched hysteresis loop at the origin

Physical Ideal Memristor

- We need to have a relation between q and ϕ
- Exists in the Josephson junction : "phase-dependent" conductance
- Description in "Introduction to Nonlinear Circuits and Networks"

Physical Non-Ideal Memristors

- Variety of examples:
  - Discharge tubes
  - pn-junctions
  - Memristors in biology

Conclusion and Q/A

- Today we discussed the memristor
- Ongoing work:
  - Work jointly done with TCNJ students (Paul B., Jake B., Matt K.):
    - Role of memristance in chaotic behaviour in the RLD (resistor-inductor-diode) circuit
    - Discharge tube memristance
  - Work done with TCNJ student Dan Funke:
    - Cardiac Memristors
- Ideal memristive behaviour in the Josephson junction
- Electromagnetic field theory for the memristor
- Questions?

Primary references:

- "Memristor - The Missing Circuit Element". Chua, L. O. IEEE Transactions on Circuit Theory. 18(5), pp. 507 - 519, 1971.
- "Memristive Devices and Systems". Chua, L. O. and Kang, S. M. Proceedings of the IEEE. 64(2), pp. 209 - 223, 1976.