Non-ideal and Ideal Memristors :

The Fourth Fundamental Circuit Element

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BS (2002), MS (2005) and PhD (2009) from the University of California, Berkeley
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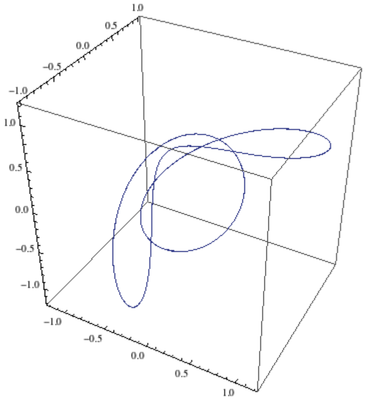
What do I work on?

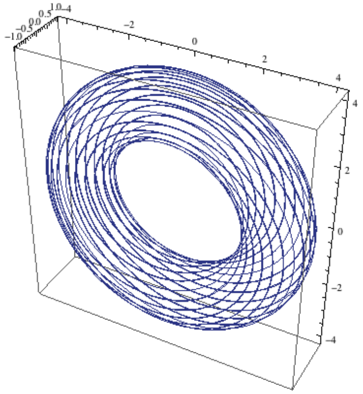
Nonlinear Dynamical Systems and Embedded Systems

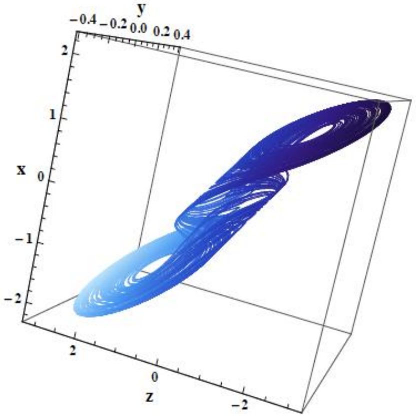
- Physical Memristors: discharge tubes, PN junctions and Josephson Junctions
  (MSOE; IIT Chennai; University of Western Australia, Perth, Australia; Vellore Institute of Technology, Vellore, India)
- Applications and Mathematical properties of the Muthuswamy-Chua system
  (MSOE; University of Western Australia, Perth, Australia)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (FPGAs)
  (MSOE; Vellore Institute of Technology; University Putra Malaysia, Malaysia)
- Pattern Recognition Using Cellular Neural Networks on FPGAs
  (MSOE; Altera Corporation)

Education

- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)







Primary Goal of this Talk

Discuss properties [2], [4] of physical memristors [4], [3], [6], [9], [10]

Outline

I. Background
  1. The Fundamental Circuit Elements
  2. The Art and Science of Device Modeling

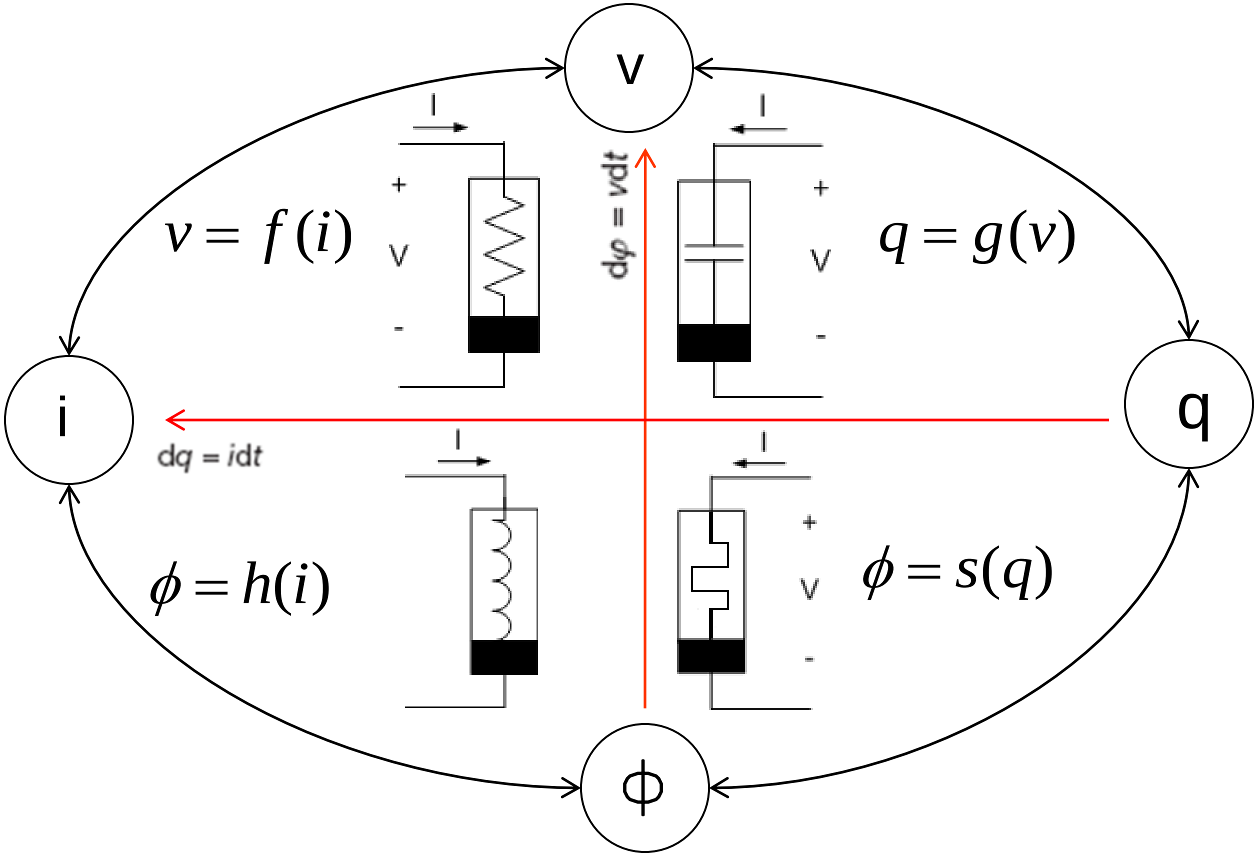
II. A “Familiar” Example – The junction diode
  1. Circuit Model

III. The Memristor
  1. Properties of the Memristor
  2. Memristive Devices
  3. Memristor Emulator

IV. Physical Memristors
  1. Non-ideal versus Ideal Memristors
  2. Non-ideal Memristors:
    a. Discharge tube
    b. Junction diode
  3. Ideal memristor:
    a. Josephson junction

V. Conclusions, Current (future) work and References

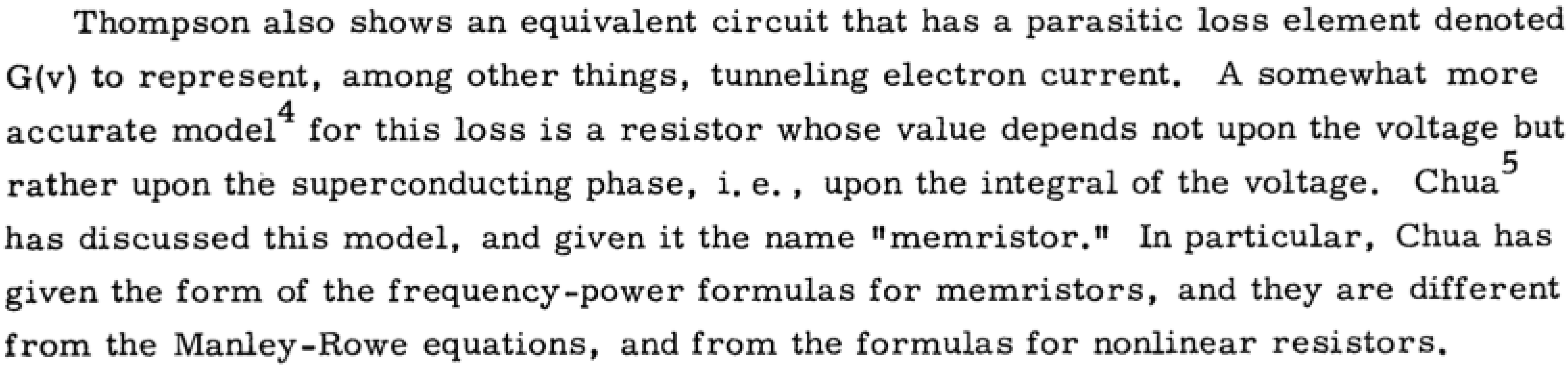
The Fundamental Circuit Elements



Memristors were first postulated by Leon. O Chua in 1971 [2]. In 2008, researchers at HP claimed to have found the “missing” memristor [11].

The Fundamental Circuit Elements (contd.)

But Paul Penfield from an MIT 1973 technical report [10], states:



Thompson also shows an equivalent circuit that has a parasitic loss element denoted G(v) to represent, among other things, tunneling electron current. A somewhat more accurate model[4] for this loss is a resistor whose value depends not upon the voltage but rather upon the superconducting phase, i. e., upon the integral of the voltage. Chua[5] has discussed this model, and given it the name "memristor." In particular, Chua has given the form of the frequency-power formulas for memristors, and they are different from the Manley-Rowe equations, and from the formulas for nonlinear resistors.

The Science and Art of Device Modeling

We first have to understand that a circuit model is not an equivalent circuit of a device since no physical device can be exactly mimicked by a circuit or mathematical model [9]. In fact, depending on the application (e.g., frequency of operation), a given device may have many distinct physical models [9]. There is no ”best model” for all occasions. The best model in a given situation is the simplest model capable of yielding realistic solutions [9]. Thus device modeling is both an art (physical device equation formulation) and science (nonlinear network synthesis).

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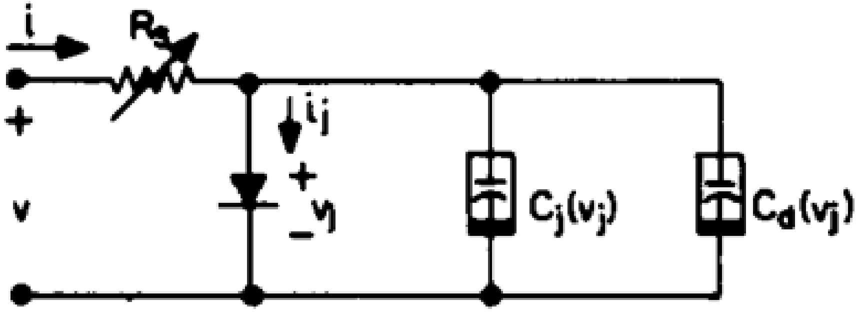
A “Familiar” Example – The junction diode: Circuit Model [3]



$R_s = R_o\, f(i_j)$ where:

$$f(i_j) \triangleq 1 - \frac{1}{X_n} \ln \frac{1 + K_r i_j}{1 + K_r i_j e^{-X_n}}$$

R_o, X_n and K_r are diode parameters

Borna–Horelick Model

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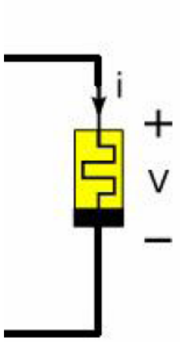
Properties of the Memristor [2] [4]

Circuit symbol:



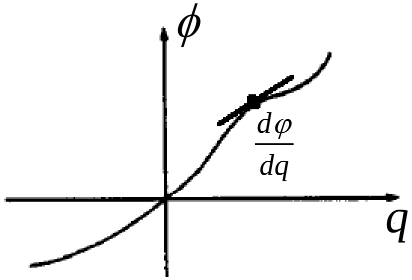
A memristor defines a *relation* of the from: $g(\phi, q) = 0 \quad (1)$

If *g* is a single-valued function of charge (flux), then the memristor is charge-controlled (flux-controlled)

Memristor i-v relationship:

M(q(t)) is the incremental memristance

$$v(t) \triangleq \frac{d\phi}{dt} = \frac{d\phi}{dq}\frac{dq}{dt} \triangleq M(q(t))i(t) \quad (2)$$



Q1: Why is the memristor called “memory resistor”?

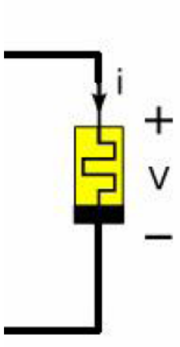
Because of the definition of memristance: $v(t) = M(q(t))i(t) = M\left(\int_{-\infty}^{t} i(\tau)\right)i(t) \quad (3)$

Q2: Why is the memristor not relevant in linear circuit theory?

1. If M(q(t)) is a constant: $v(t) = M(q(t))i(t) = Mi(t) = Ri(t) \quad (4)$
2. Principle of superposition is not* applicable. Proof:

$$M\left(\int_{-\infty}^{t} (i_1 + i_2)(\tau)\right)(i_1 + i_2)(t) = M\left(\int_{-\infty}^{t} (i_1)(\tau) + \int_{-\infty}^{t} (i_1)(\tau)\right)(i_1 + i_2)(t) \neq M\left(\int_{-\infty}^{t} (i_1)(\tau)\right)i_1(t) + M\left(\int_{-\infty}^{t} (i_2)(\tau)\right)i_2(t)$$

Memristive Devices [4]



$$\begin{aligned} v &\triangleq R(\vec{z},i)i \\ \dot{\vec{z}} &= f(\vec{z},i) \end{aligned} \qquad (5)$$

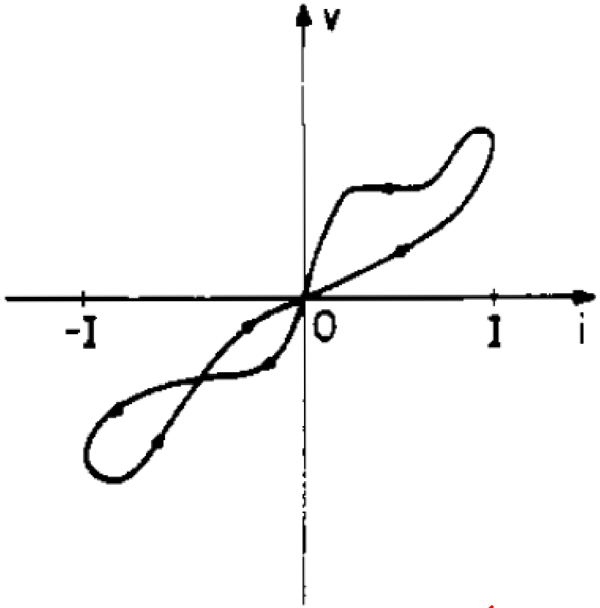
$$\xrightarrow{\vec{z} \triangleq q,\, R(\vec{z},i) \triangleq M(q)}$$

$$\begin{aligned} v &\triangleq M(q)i \\ \dot{q} &= i \end{aligned} \qquad (6)$$

The functions *R* and *f* are defined as:

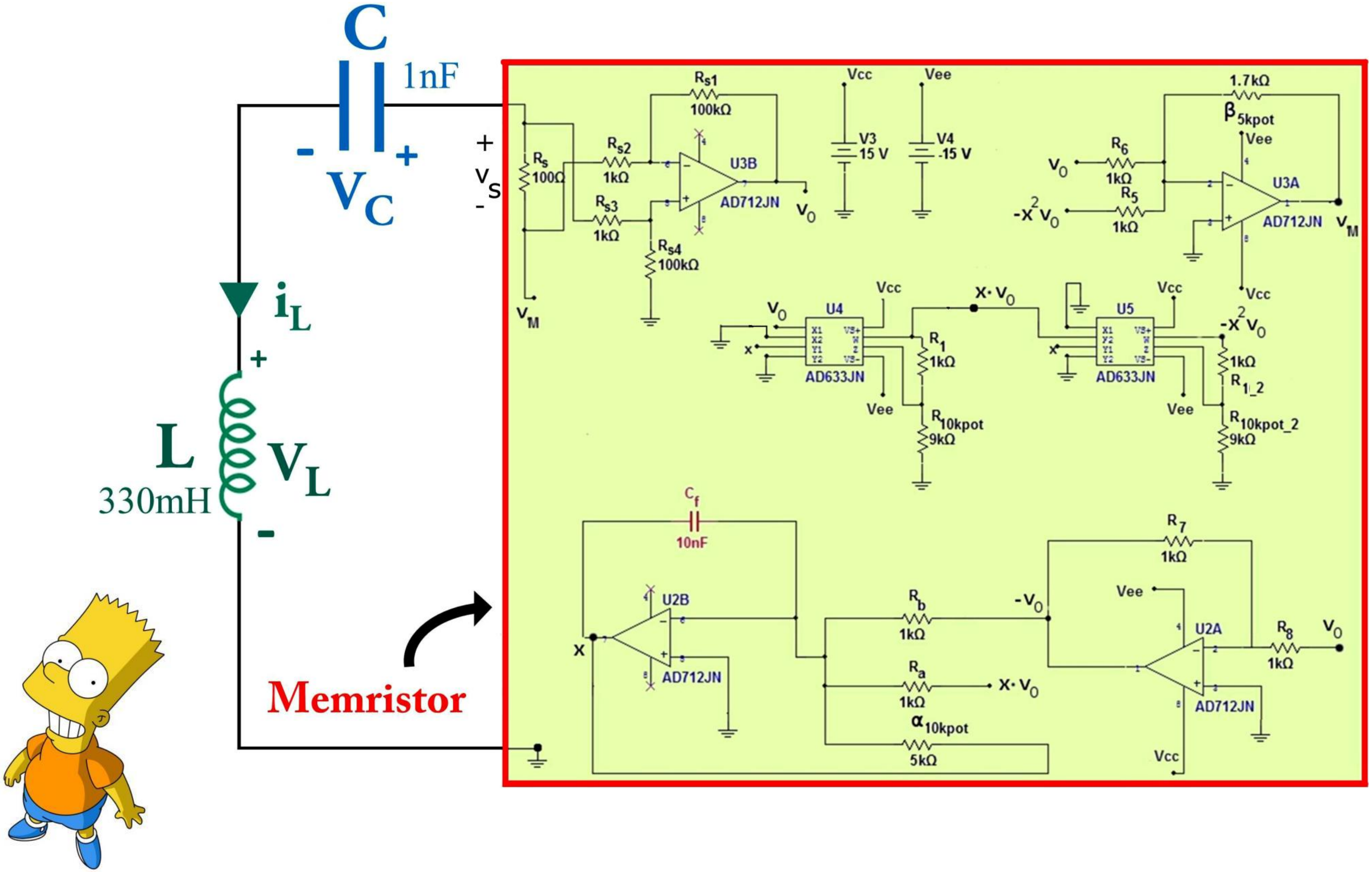
$$R : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$$

$$f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$$

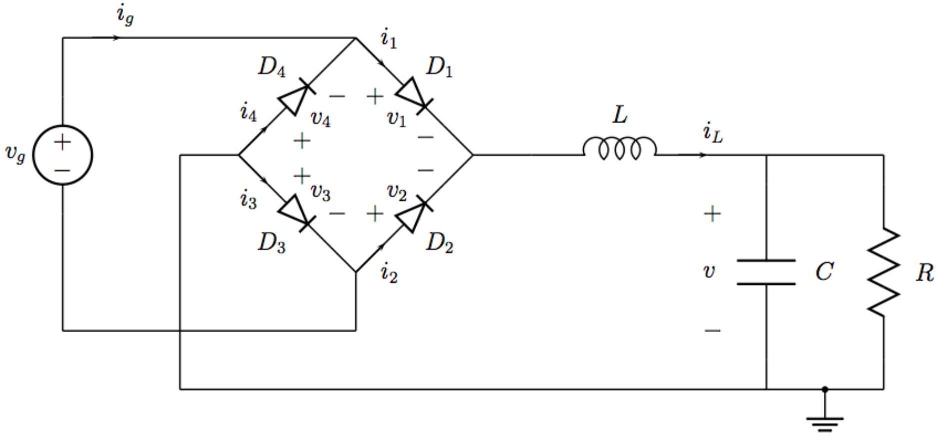


In Eq. (5), $R(\vec{z},i) \not\to \infty$

Memristor Emulation [8]



Memristor Emulation : Passive Elements only [5]



$$v_1 = nV_T \ln\left(\frac{i_L + 2I_S}{2I_S \exp\left(-\frac{v_g}{2nV_T}\right)\cosh\left(\frac{v_g}{2nV_T}\right)}\right) \qquad (7)$$

$$i_g = (i_L + 2I_S)\tanh\left(\frac{v_g}{2nV_T}\right) \qquad (8)$$

$x_1 = v\,(V_T)^{-1}$

$x_2 = i_L\,(I_S)^{-1}$

$u = v_g\,(V_T)^{-1}$

$y = i_g\,(I_S)^{-1}$

$\tau = t\,(t_0)^{-1}$

$t_0 = 2\pi\,(\omega_0)^{-1}$

$\omega_0 = [(LC)^{-1} - (RC)]^{\frac{1}{2}}$

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}, u, t)$$

$$y = g(\mathbf{x}, u, t)u \qquad (9)$$

$$f(\mathbf{x}, u, t) = \begin{bmatrix} \beta(x_2 - \alpha x_1) \\ \gamma\left(u - x_1 - 2\ln\left(\frac{x_2+2}{2\exp\left(-\frac{u}{2n}\right)\cosh\left(\frac{u}{2n}\right)}\right)\right) \end{bmatrix} \qquad (10)$$

$$g(\mathbf{x}, u, t) = (x_2 + 2)\frac{\sum_{m=0}^{\infty} \frac{\left(\frac{u}{2n}\right)^{2m}}{(2m+1)!}}{\sum_{m=0}^{\infty} \frac{\left(\frac{u}{2n}\right)^{2m}}{(2m)!}}, \qquad (11)$$

Memristor Emulators – Pinched Hysteresis Loops [8] [5]

From [3], pinched-hysteresis for 3 kHz sinusoidal input

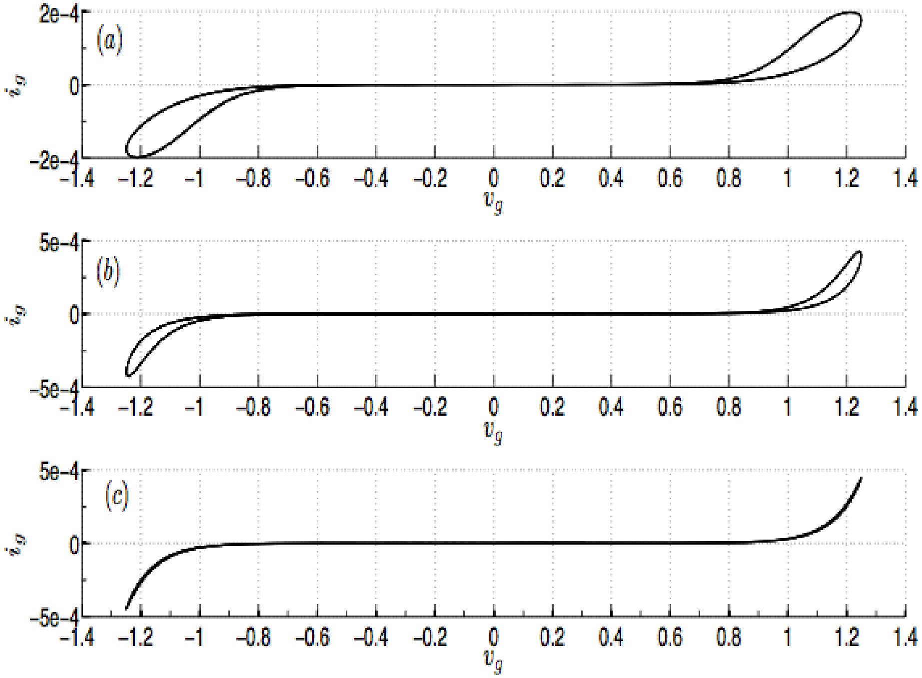
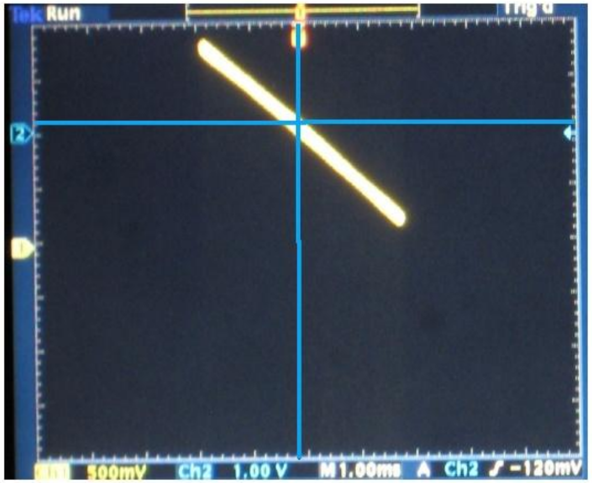


Fig. 3 *Current-voltage characteristics observed in numerical simulations of the mathematical model of the proposed circuit for a sine-wave input with* f *set to* 10 *(plot (a)),* 100 *(plot (b)) and* 1000 Hz *(plot (c)).*

From [4]



From [3], pinched-hysteresis for 35 kHz sinusoidal input

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Non-ideal versus Ideal Memristors

Definition:

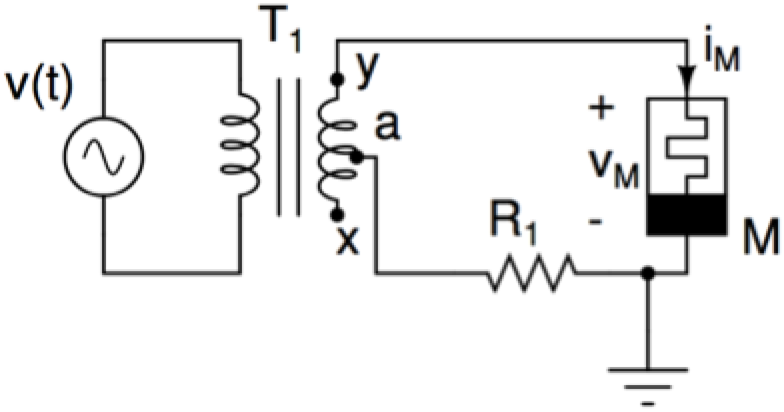
A memristor that establishes a relation between charge and flux is an ideal memristor. Any other memristor (memristive device) is non-ideal.

Non-ideal Memristor : Discharge Tube [9]

$$v = M(n)i$$

$$\dot{n} = -\beta n + \alpha M(n) i^2 \qquad (12)$$

$$M(n) \triangleq \frac{F}{n} \qquad (13)$$



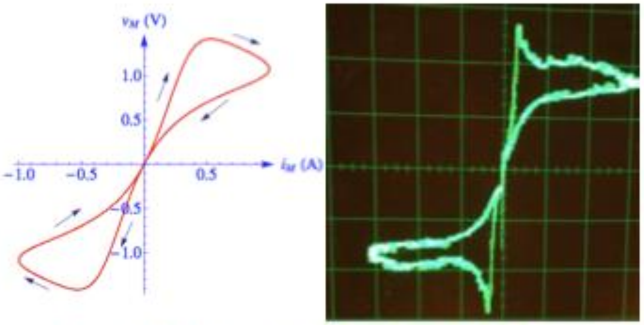


Figure 3. Simulation versus experimental result for memristor pinched-hysteresis (Lissajous) figure. v_M, i_M are indicated on the plot. Parameters used for simulation: $\beta = 0.1, \alpha = 0.1, F = 1, \omega = 0.063$ The discharge tube is a Phillips 15 W F15T8.

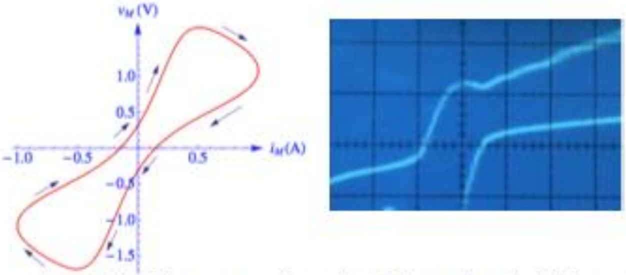


Figure 4. Simulation versus experimental result for memristor pinched-hysteresis (Lissajous) figure. For simulation, we used a 5 H inductor. For the physical setup, we used a 300 H inductor and "zoomed-in" at the origin since the transformer has a measured secondary inductance of 1400 H at 60 Hz.

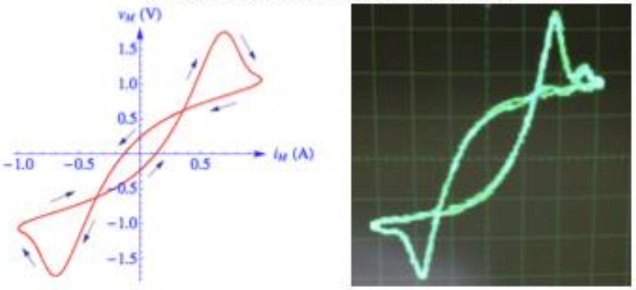
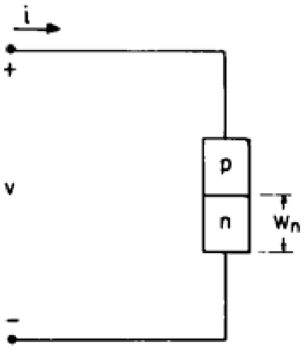
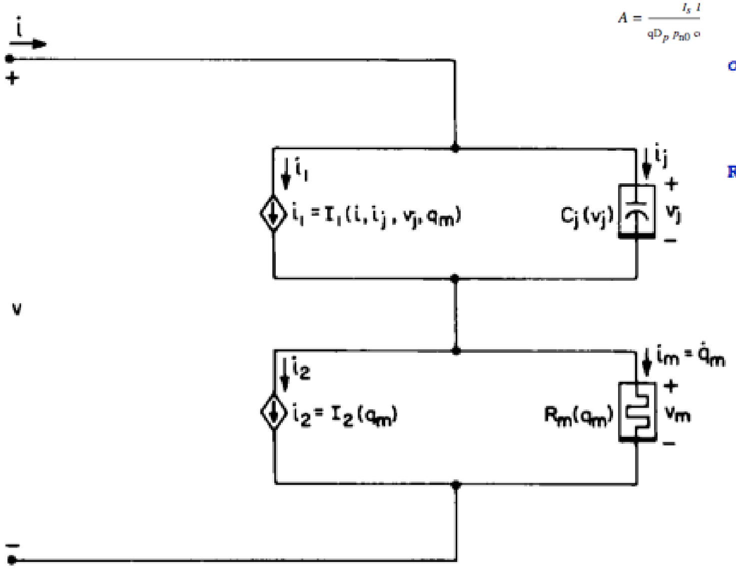


Figure 5. Simulation versus experimental result for memristor pinched-hysteresis (Lissajous) figure. For simulation, we used a 1 F capacitor; for the physical experiment, we show a 100 nF capacitor in parallel.

Non-ideal Memristor : Junction Diode [3]



(a)



(b)

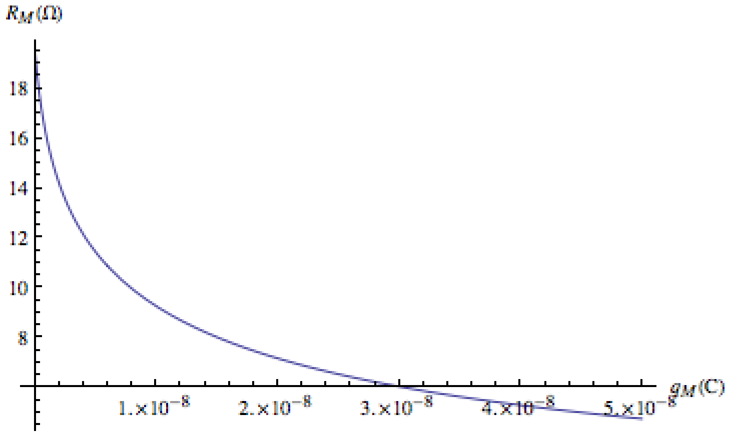
Simulation of $R_m(q_m)$ for memristive model of a pn junction diode (Ref. : A Memristive Circuit Model for P-N Junction Diodes, Chua, L. O. and Tseng, Chong-Wei. International Journal of Circuit Theory and Applications, Vol. 2, 367-389 (1974)

Parameters :

$N_D = 10^{15}\ \mathrm{cm}^{-3}$ (donor concentration)
$\mu_n = 1350\ \mathrm{cm}^2 / V.\mathrm{sec}$ (electron mobility)
$\mu_p = 480\ \mathrm{cm}^2 / V.\mathrm{sec}$ (hole mobility)
$I_S = 0.5 * 10^{-12}\ A$ (diode saturation current)
$\tau_p = 10^{-7}$ sec (hole recombination life time)
$V_T = 26.047$ mV (thermal voltage)
$D_p = \mu_p V_T$ (hole diffusion constant)
$L_p = \sqrt{D_p \tau_p}$ (hole diffusion length)
$w_n = 5 L_p$ (width of the n – type region or base width)
$T = 300\ K$ (Ambient temperature)
$\psi_0 = 0.9\ V$ (barrier potential)
$n_{n0} = N_D$ (approximately) (equilibrium electron concentration in the n – type region)
$n_i = 8.367 * 10^8\ \mathrm{cm}^{-3}$ (@ 300 K) (intrinsic concentration)
$p_{n0} = \frac{(n_i)^2\ 300}{n_{n0}} = 2.1 * 10^5$ (equilibrium hole concentration in the p – type region)
$A = \frac{I_s\ l}{qD_p\ p_{n0}\ o}$

$$\sigma[x\_,\ qm\_] := q * \mu_n * n_{n0} + q * \mu_p * \left(p_{n0} + \frac{qm}{A * q * L_p} * \left(\frac{\mathrm{Sinh}\left[\frac{W_n}{L_p}\right]}{\mathrm{Cosh}\left[\frac{W_n}{L_p}\right] - 1} \right) * \left(\mathrm{Cosh}\left[\frac{x}{L_p}\right] - \mathrm{Coth}\left[\frac{W_n}{L_p}\right] * \mathrm{Sinh}\left[\frac{x}{L_p}\right] \right) \right) \tag{14}$$

$$\mathrm{Rm}[qm\_] := \frac{1}{A} * \int_0^{W_n} \frac{1}{\sigma[x,\ qm]}\, dx \tag{15}$$



Ideal Memristor : Josephson Junction [6] [10]

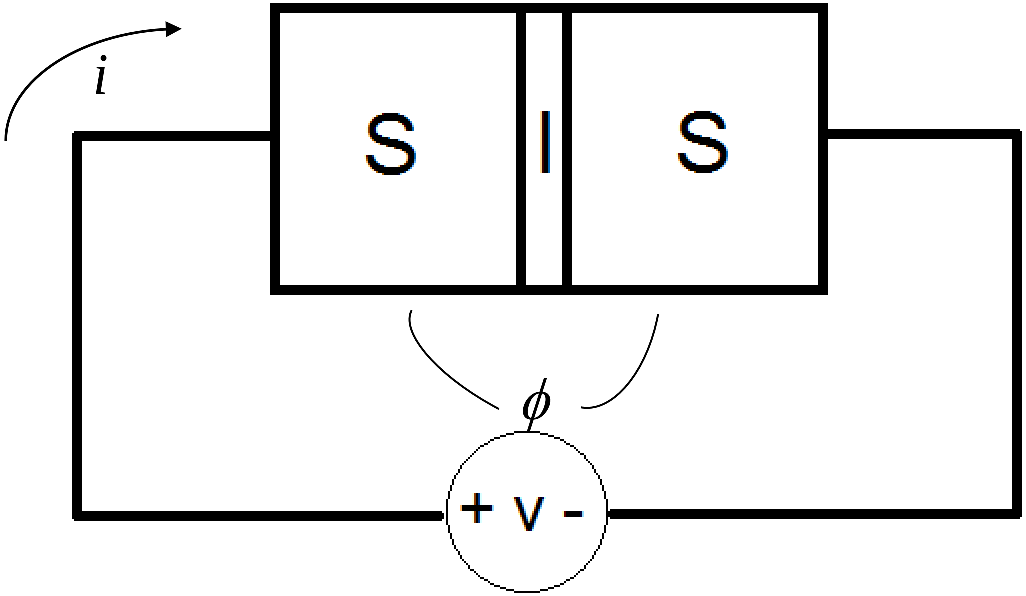
$$E = h \cdot \upsilon \quad (16)$$

$$= h\frac{\omega}{2\pi}$$

$$= \hbar\frac{d\phi}{dt}$$

$$2e^{-}\mathrm{v} = \hbar\frac{d\phi}{dt}$$

$$\frac{2e^{-}}{\hbar}\mathrm{v} = \frac{d\phi}{dt}$$



$$\frac{d\phi}{dt} = \frac{2e^{-}}{\hbar}\mathrm{v} \quad (17)$$

Ideal Memristor : Josephson Junction (contd.)

Suppose v = 1 uV. f (in Hz) for the Josephson junction ≈ 0.482 GHz!

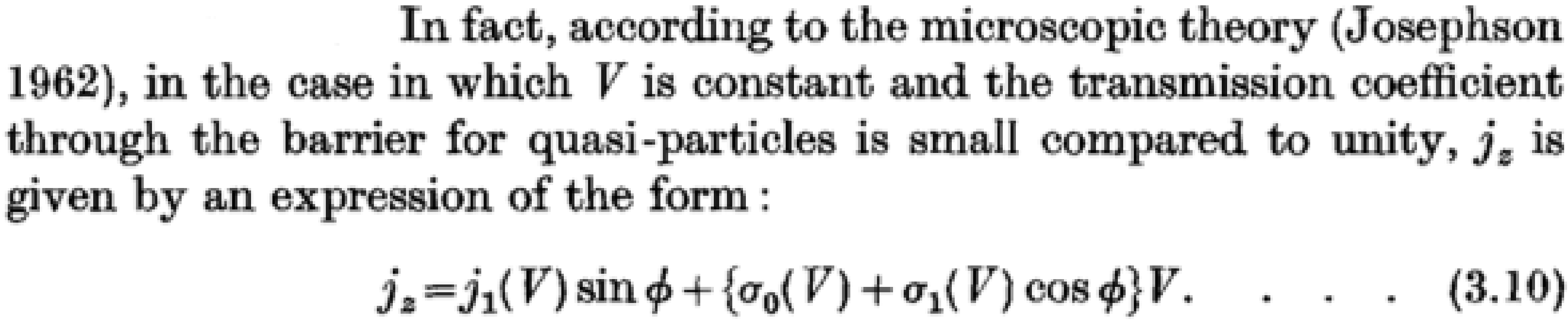
$$\frac{d\phi}{dt} = \frac{2e^-}{\hbar}(1\ \mu V)$$

$$\Rightarrow f = \frac{\frac{2e^-}{\hbar}(1\ \mu V)}{2\pi} \approx 482 \text{ MHz}$$

$$\frac{d\Phi_B}{dt} = \text{v}$$

$$\Phi_B \triangleq \frac{\hbar}{2e^-}\phi = k\phi \qquad (18)$$

Ideal Memristor : Josephson Junction (contd.)

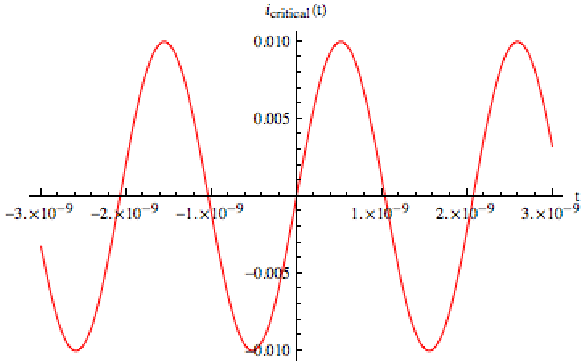


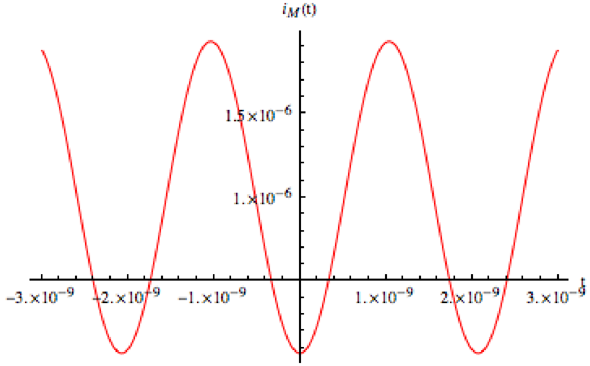
In fact, according to the microscopic theory (Josephson 1962), in the case in which V is constant and the transmission coefficient through the barrier for quasi-particles is small compared to unity, j_z is given by an expression of the form :

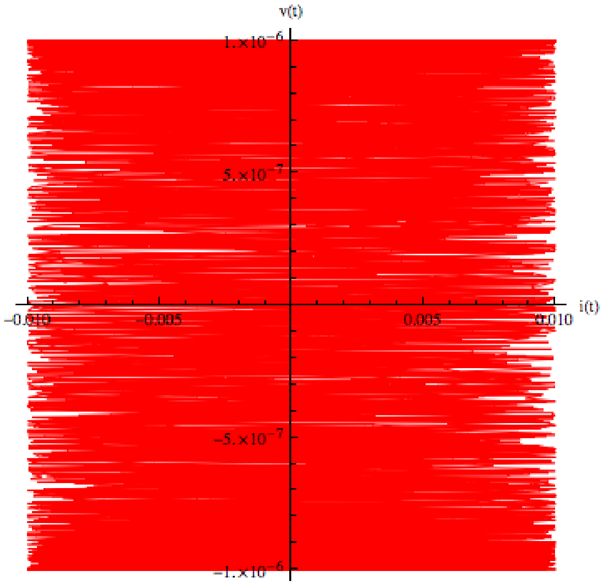
$$j_z = j_1(V)\sin\phi + \{\sigma_0(V) + \sigma_1(V)\cos\phi\}V. \quad . \quad . \quad . \quad (3.10)$$

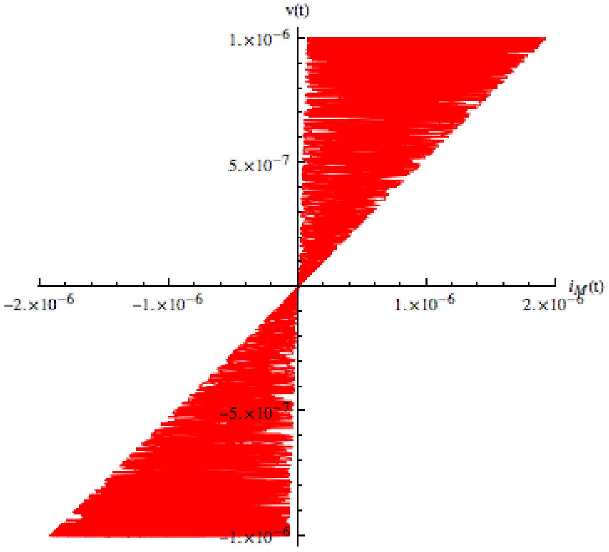
$$I = I(\mathrm{v})\sin\left(\frac{2e^-}{\hbar}\Phi_{\mathrm{B}}\right) + \left(\sigma_0(\mathrm{v}) + \sigma_1(\mathrm{v})\cos\left(\frac{2e^-}{\hbar}\Phi_{\mathrm{B}}\right)\right)\mathrm{v} \qquad (19)$$

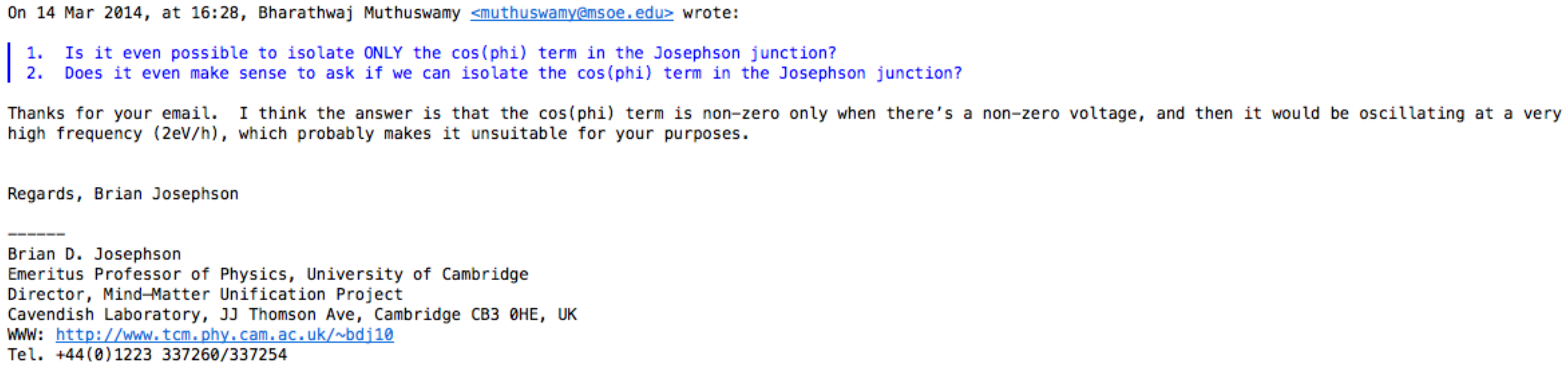
Ideal Memristor : Josephson Junction (contd.)











On 14 Mar 2014, at 16:28, Bharathwaj Muthuswamy <muthuswamy@msoe.edu> wrote:

> 1. Is it even possible to isolate ONLY the cos(phi) term in the Josephson junction?
> 2. Does it even make sense to ask if we can isolate the cos(phi) term in the Josephson junction?

Thanks for your email. I think the answer is that the cos(phi) term is non-zero only when there's a non-zero voltage, and then it would be oscillating at a very high frequency (2eV/h), which probably makes it unsuitable for your purposes.

Regards, Brian Josephson

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Conclusions

- The memristor is a fundamental circuit element.
- There are physical (electronic) devices that display memristive effects.
- The memristor is a “nonlinear” device (unlike the resistor, capacitor and inductor).

Current and Future Work

1. Understanding v-i characteristics of discharge tube: Circuit for plotting v-i (with Dr. Iu (UWA), Dr. Loo (HKP))
2. Chaos in Muthuswamy-Chua with discharge tube [1] (with Dr. Iu, Dr. Loo and Dr. Corinto (Politecnico di Torino))
3. Identifying ideal memristive behavior in the Josephson junction (with IIT-Chennai and VIT)
4. Complete SPICE model of junction diode with memristor (with Dr. Jevtic (MSOE))
    - Specifically for 3. and 4., use the idea of frequency-power formulae?
5. An electromagnetic field (physical) theory for memristors (with Dr. Jevtic and Dr. Thomas (MSOE)).

References

1. Braun, T. et. al. “Observation of Deterministic Chaos in Electrical Discharges in Gases”, *Physical Review Letters,* Vol. 59, No. 6, pp. 613-616, 1987.
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Questions and Discussion…