Chaotic Circuits – An Introduction

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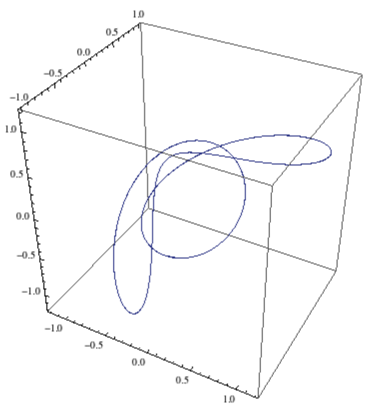
What do I work on?

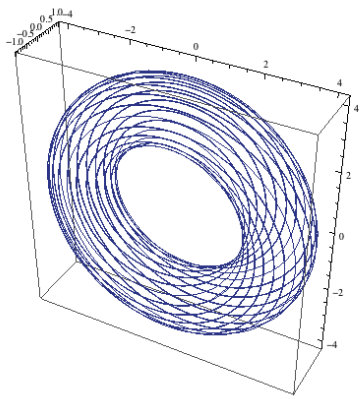
Nonlinear Dynamical Systems and Embedded Systems

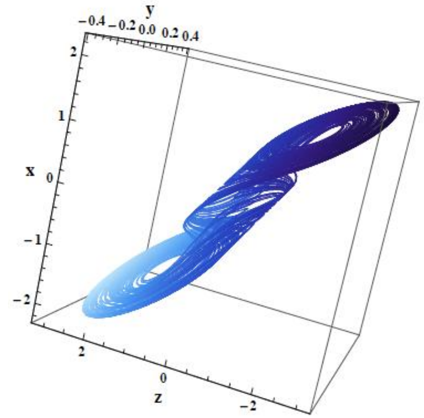
- Applications and Mathematical properties of the LCM chaotic circuit
    - Number Theory (Bharathidasan University, Tiruchy, India)
    - Local activity (University of Western Australia, Perth, Australia)
    - Flow manifolds (I.U.T. de Toulon, La Garde Cedex, France)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (University Putra Malaysia, Malaysia)
- Pattern Recognition Using Cellular Neural Networks (University of California, Berkeley, USA)
- Gait generation using nonlinear dynamics for children with Cerebral Palsy* (Medical College of Wisconsin, Wauwutosa, USA)
- Memristive behavior in superconductors (University of California, Berkeley, USA; Vellore Institute of Technology, Vellore, India)

Education

- edX program (University of California, Berkeley; Massachusetts Institute of Technology; Harvard University, USA)
- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)







This talk : LCM Chaotic Circuit

Outline

I. Prerequisites for understanding this talk:
1. First course in circuit theory*
2. First course in differential equations

II. Introduction
1. Fundamental Circuit Theory [2] [3]
2. Static vs. Dynamical systems

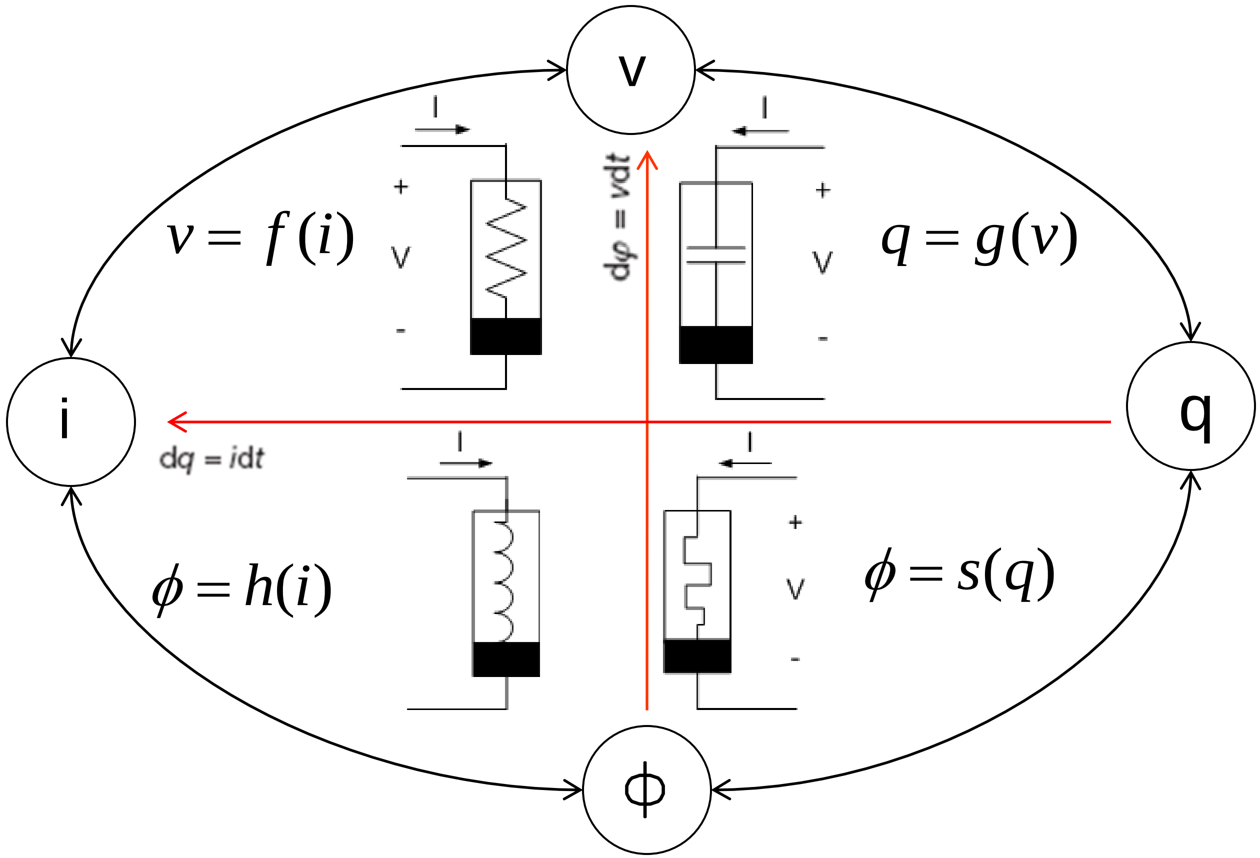
III. Steady-state Solutions of Differential equations
1. Simple Harmonic Oscillator
2. Quasi-periodicity
3. Chaos [1] [5] [10]

IV. Physical Realization - electronic circuits
1. LCR circuit
2. LCM circuit [7]
3. Mathematical Property of chaos - The "Dimension" of a chaotic attractor [9]
4. Mathematical Property of chaos - The Frequency Spectrum [7]

V. An Application of Chaos : Human arrhythmia control [4]

VI. References

Introduction : Fundamental Circuit Theory [2] [3]

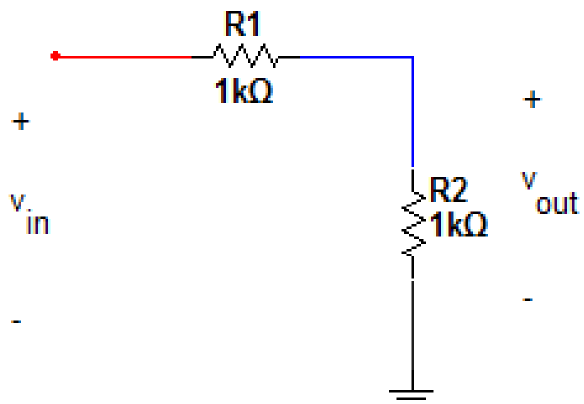


Memristors were first postulated by Leon. O Chua in 1971 [2]

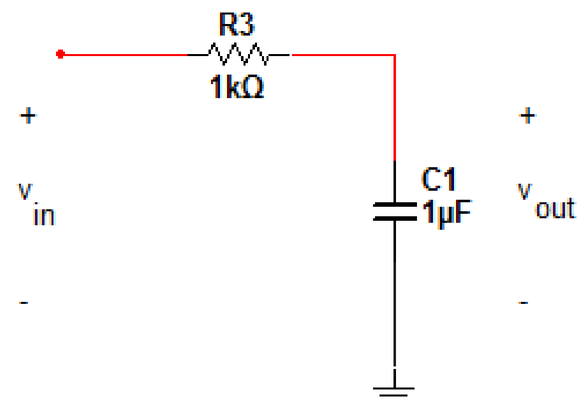
Introduction :
Static vs. Dynamical Systems

1. Mathematical definition of a system

$$y(t) = S(x)(t) \quad y, x : \mathbb{R} \to \mathbb{R}, t \in \mathbb{R} \qquad (1)$$



$$v_{out}(t) = \frac{R2}{R1+R2} v_{in}(t) \qquad (2)$$



$$R3C1 \frac{dv_{out}}{dt} + v_{out} = v_{in} \qquad (3)$$

$$v_{out}(t) = v_{out}(0) e^{-t/(R3C1)} + \frac{1}{R3C1} \int_0^t e^{\frac{-(t-\tau)}{R3C1}} v_{in}(\tau) d\tau \qquad (4)$$

2. Concept of a linear time-invariant system

3. Various system behaviors: stable, unstable

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Simple Harmonic Oscillator

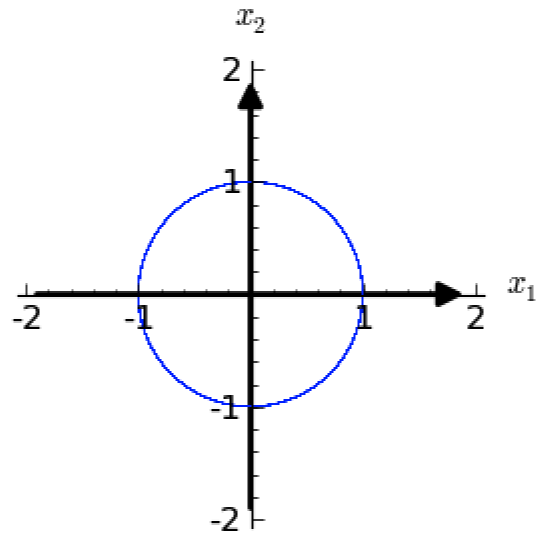
$$\ddot{x} + x = 0 \qquad (5)$$

$$\downarrow \; (x_1 \triangleq x, x_2 = \dot{x})$$

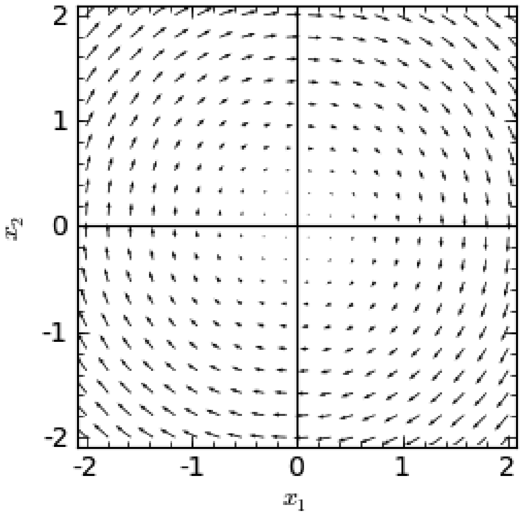
$$\dot{x}_1 = x_2 \qquad (6)$$

$$\dot{x}_2 = -x_1$$

Phase portrait:



Vector field:



Plots were obtained using SAGE:
http://www.sagemath.org/index.html

Quasi-Periodicity

After stable, unstable and oscillatory behavior, we have quasi-periodicity [6]

$$\ddot{x} - (\lambda + z + x^2 - \frac{1}{2}x^4)\dot{x} + {\omega_0}^2 x = 0 \quad (7)$$

$$\dot{z} = \mu - x^2$$

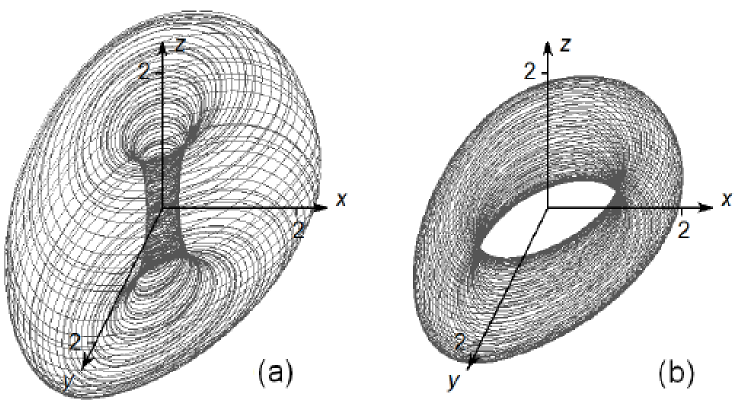


Figure 2. Portraits of attractor in the three-dimensional phase space of variables $(x,\ y = \dot{x}/\omega_0,\ z)$ for the model (1) with λ=0, $\omega_0 = 2\pi$ at μ=0.5 (a) and μ=0.9 (b)

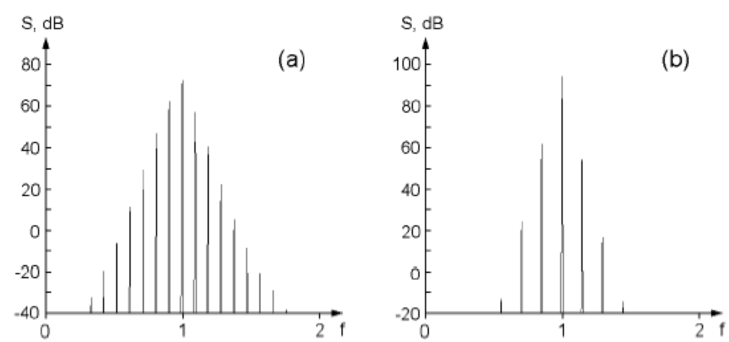
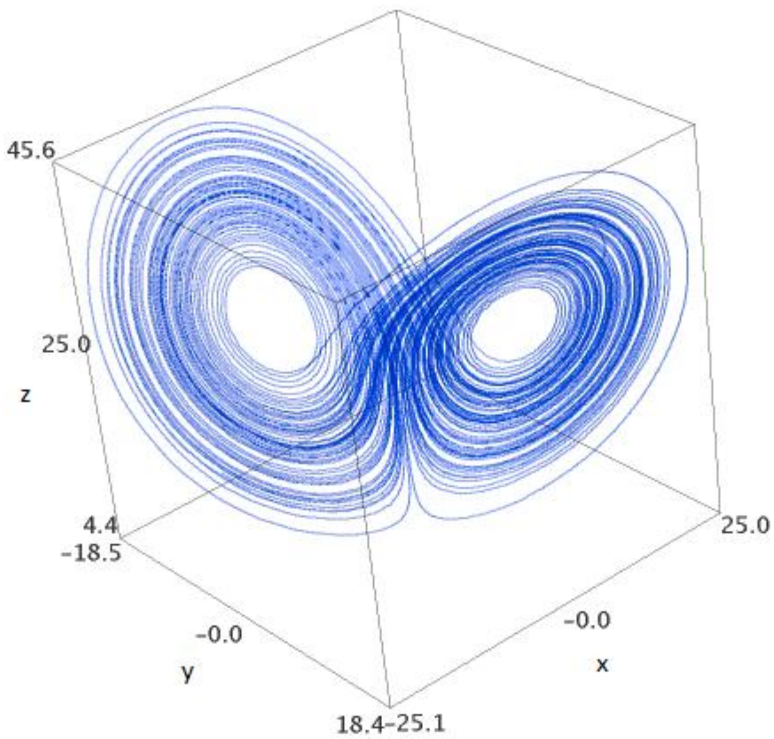


Figure 3. Fourier spectra of oscillations of the variable x on the attractor for the model (1) with λ=0, $\omega_0 = 2\pi$ at μ=0.5 (a) and μ=0.9 (b)

Chaotic Systems [1] [5] [10]

- "Birth" of Chaos: Lorenz Attractor [8]
  - Edward Lorenz introduced the following nonlinear system of differential equations as a crude model of weather in 1963:



$$\dot{x} = -\sigma \cdot x + \sigma \cdot y$$
$$\dot{y} = \rho \cdot x - y - x \cdot z \qquad (8)$$
$$\dot{z} = -\beta \cdot z + x \cdot y$$

Parameters: $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

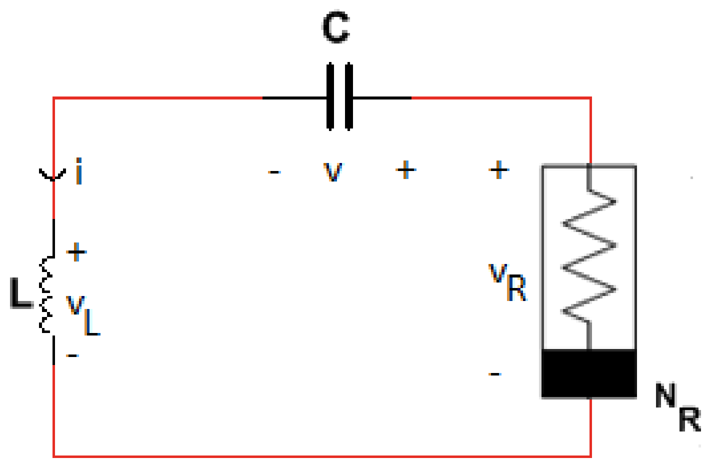
ICs: $x_0 = 10, y_0 = 20, z_0 = 30,$

Simulation time: 100 seconds

  - Lorenz discovered that model dynamics were extremely sensitive to initial conditions and the trajectories were aperiodic but bounded.
  - But, does chaos exist *physically*? Answer is: YES.

Outline

LCR Circuit - Derivation of Circuit Equations



Circuit equations:

$$\dot{v} = \frac{i}{C} \qquad (9)$$

$$i' = \frac{v_L}{L} \qquad (10)$$

$$v_R = -iR \qquad (11)$$

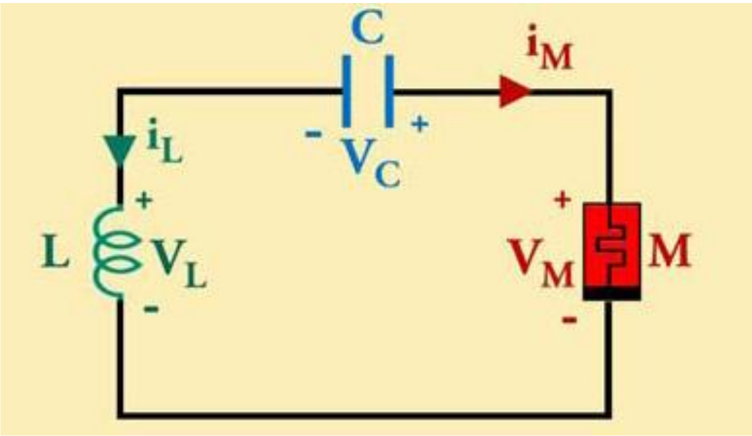
ONE differential equation – start with KVL: $v_R - v - v_L = 0 \qquad (12)$

$$\downarrow ?$$

$$i'R + \frac{i}{C} + Li'' = 0 \qquad (13)$$

LCM Circuit - Derivation of Circuit Equations [7]

$$v_M \triangleq R(z, i_M) i_M$$
$$\dot{z} = f(z, i_M)$$



Circuit equations:

$$\dot{v}_c = \frac{i_L}{C}$$
$$i'_L = \frac{-1}{L}\left(v_C + R(z, i_L) i_L\right)$$
$$\dot{z} \triangleq f(z, i_L)$$

$$\xrightarrow{x \triangleq v_c,\, y \triangleq i_L}$$

System equations:

$$\dot{x} = \frac{y}{C}$$
$$\dot{y} = \frac{-1}{L}\left(x + R(z, y) y\right) \quad (14)$$
$$\dot{z} = f(z, y)$$

Specifically:

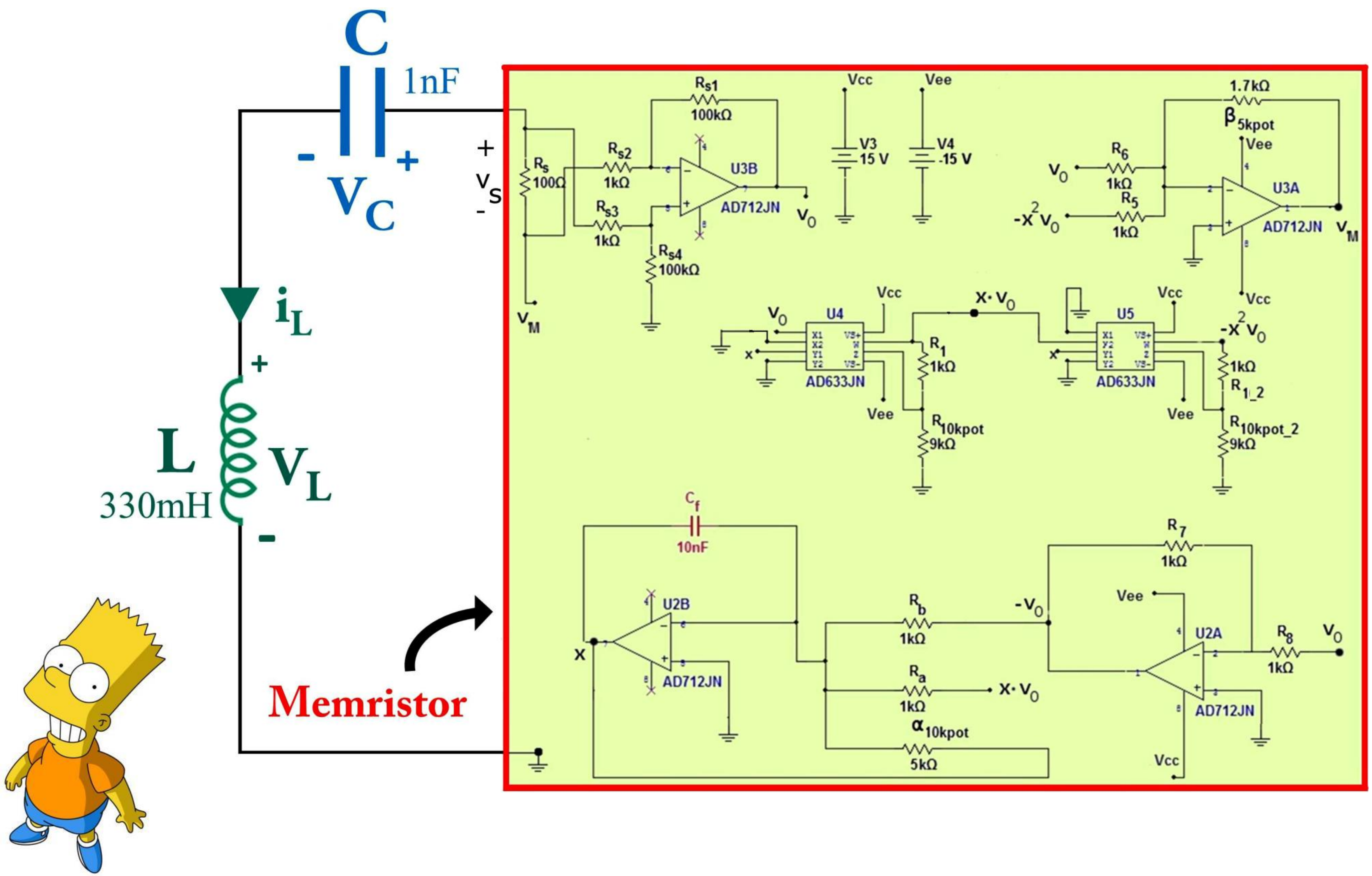
$$\dot{x} = \frac{y}{C}$$
$$\dot{y} = \frac{-1}{L}\left(x + \beta(z^2 - 1) y\right) \quad (15)$$
$$\dot{z} = -y - \alpha z + yz$$

Parameters:

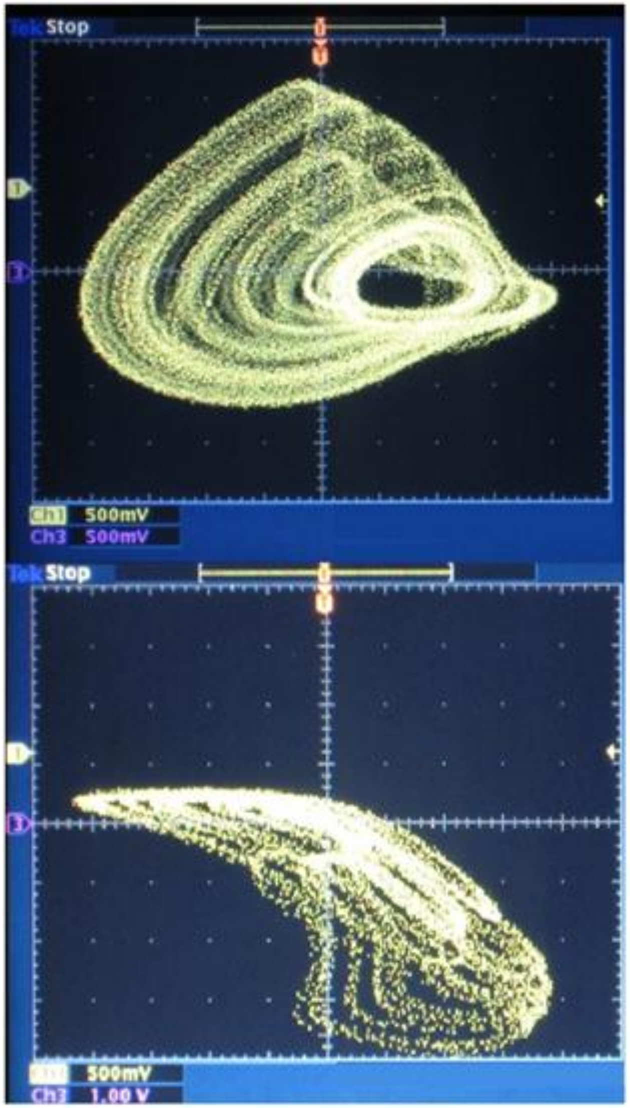
$$\xrightarrow[C=1, L=3]{}$$
$$\beta = \frac{3}{2}, \alpha = \frac{3}{5}$$

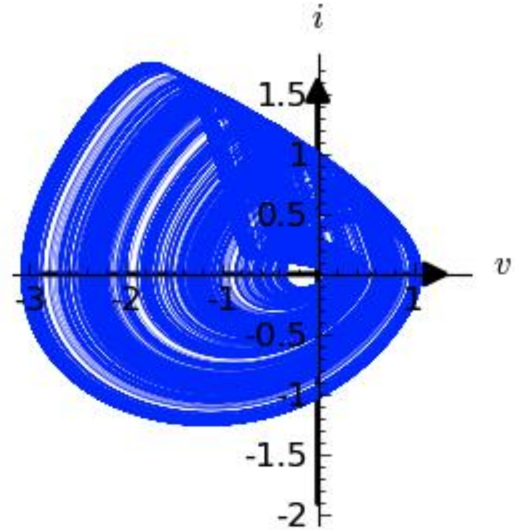
$$\dot{x} = y$$
$$\dot{y} = \frac{-x}{3} - \frac{z^2 y}{2} + \frac{y}{2} \quad (16)$$
$$\dot{z} = -y - 0.6z + yz$$

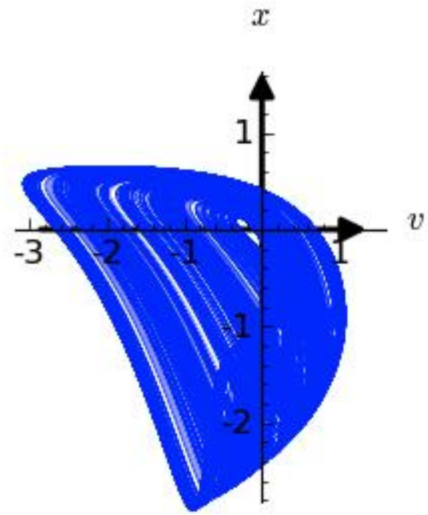
LCM Circuit - Physical Realization [7]



Attractors from the Circuit [7]



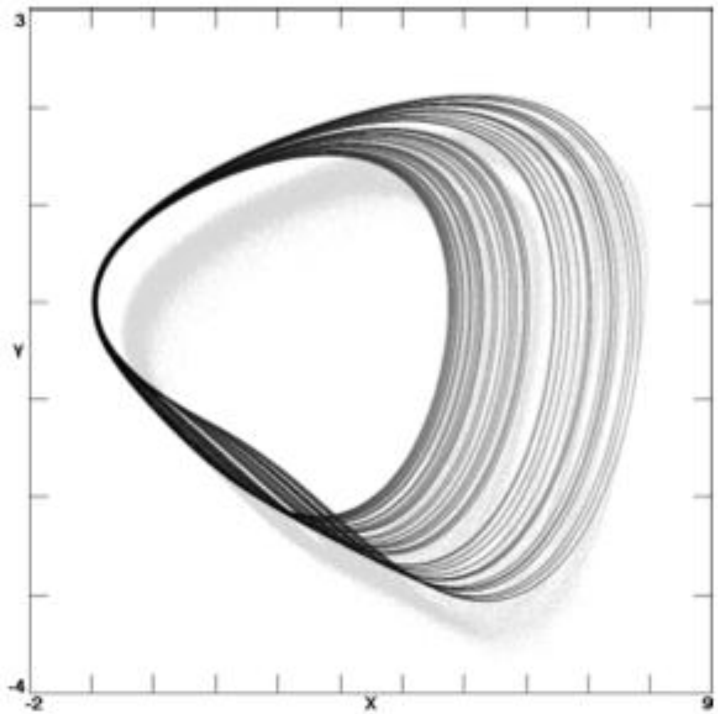




Mathematical Property of Chaos - “Dimension” of a Chaotic Attractor [9]

$$\dddot{x} + \alpha \ddot{x} + x + f(\dot{x}) = 0 \quad (17)$$

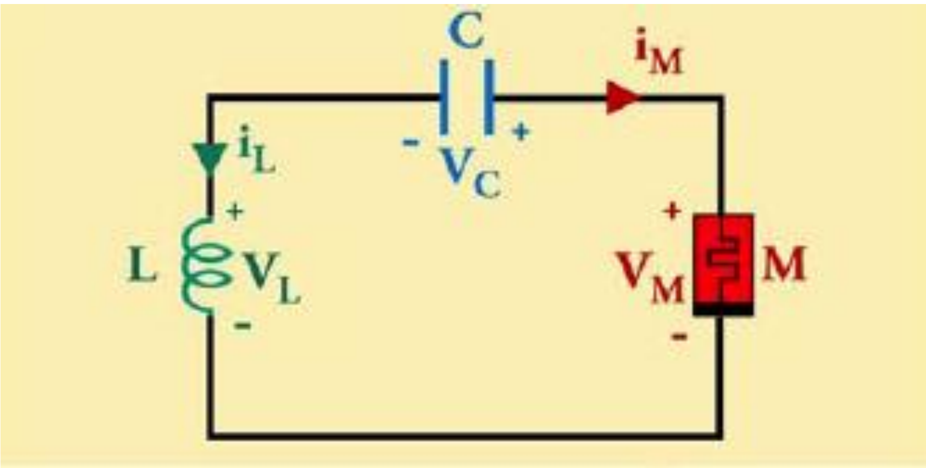
$$\alpha = 2.02,\ f(\dot{x}) = -\dot{x}^2, \text{i.c=(5,2,0)}$$

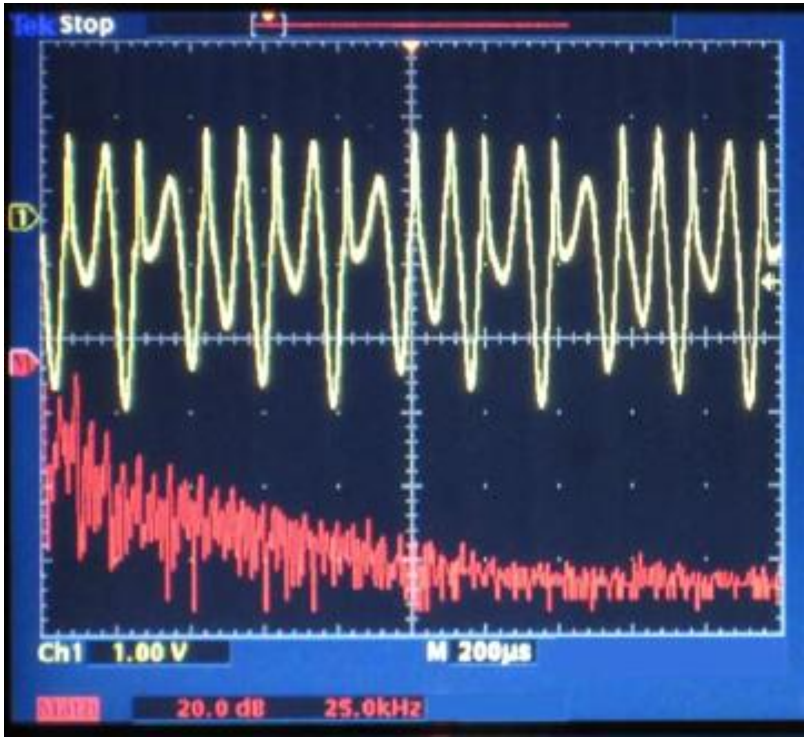


λ=(0.0486,0,-2.0686)

$$D_{KY} = 2 + \frac{0.0486 + 0}{|-2.0686|} \approx 2.02349$$

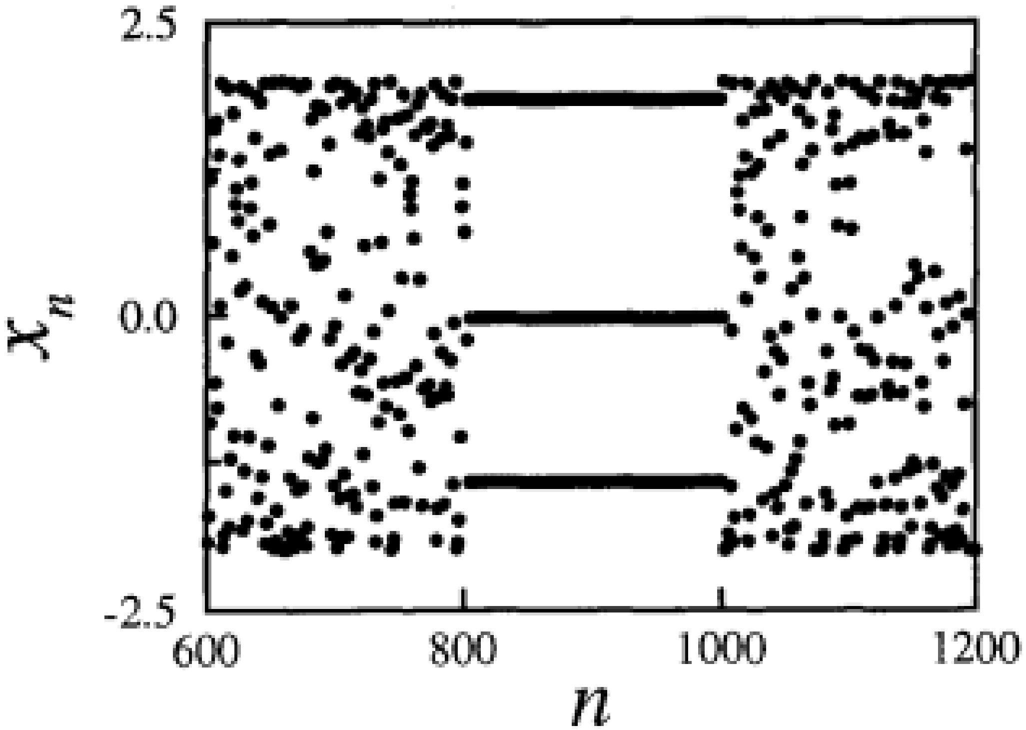
Mathematical Property of Chaos - The Frequency Spectrum [7]





An Application of Chaos: Human arrhythmia control [4]

1. Arrhythmia = “not in rhythm” = bad



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Questions?

Now….Computer Science ☺ - SICP!