Part 1* - Positive Feedback and Relaxation Oscillators

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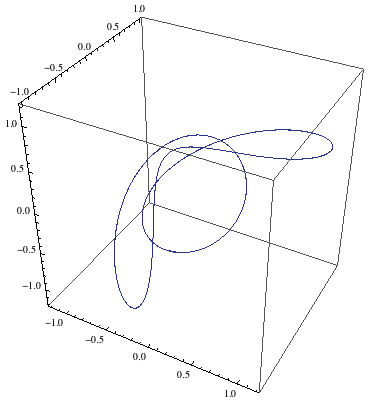
What do I work on?

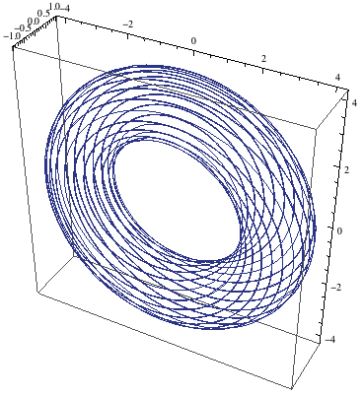
Nonlinear Dynamical Systems and Embedded Systems

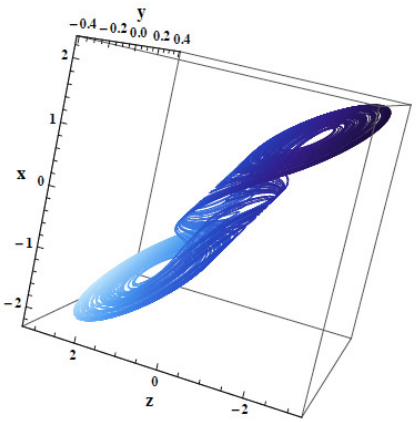
- Physical Memristors: discharge tubes, PN junctions and Josephson Junctions
  (MSOE; IIT Chennai; Vellore Institute of Technology)
- Applications and Mathematical properties of the Muthuswamy-Chua system
  (MSOE; Vellore Institute of Technology; University Putra Malaysia, Malaysia)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (FPGAs)
  (MSOE; Vellore Institute of Technology; University Putra Malaysia, Malaysia)
- Pattern Recognition Using Cellular Neural Networks on FPGAs
  (MSOE; Vellore Institute of Technology; University of California, Berkeley; Altera Corporation)

Education

- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)







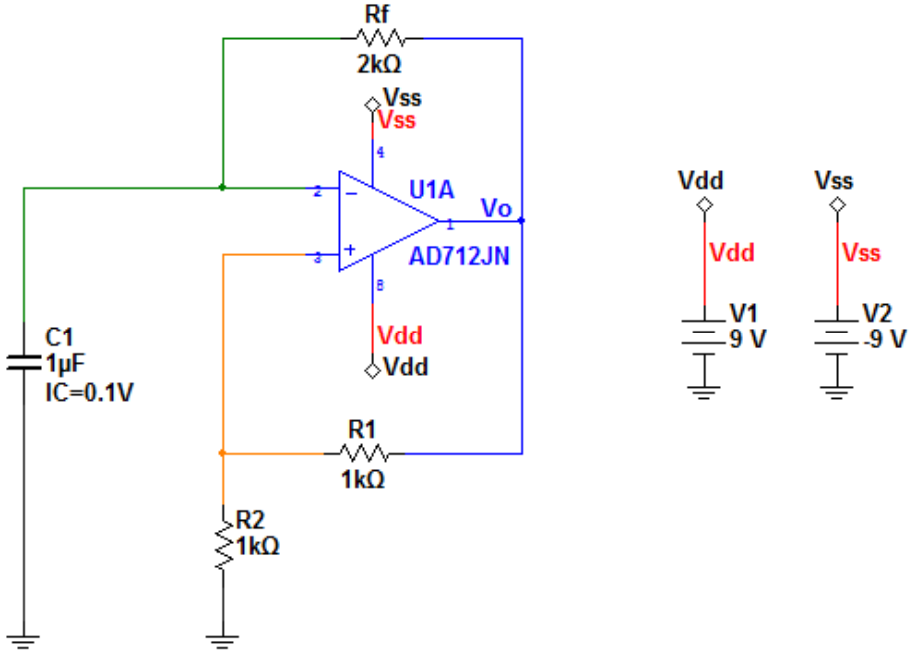
Professor : What are the applications of this research?

My answer : To quote Max Planck, “Scientific discoveries and inventions have been achieved only by those who went in pursuit of them without any applications in mind.”

My question : Who is Max Planck?

Goal of this Lecture

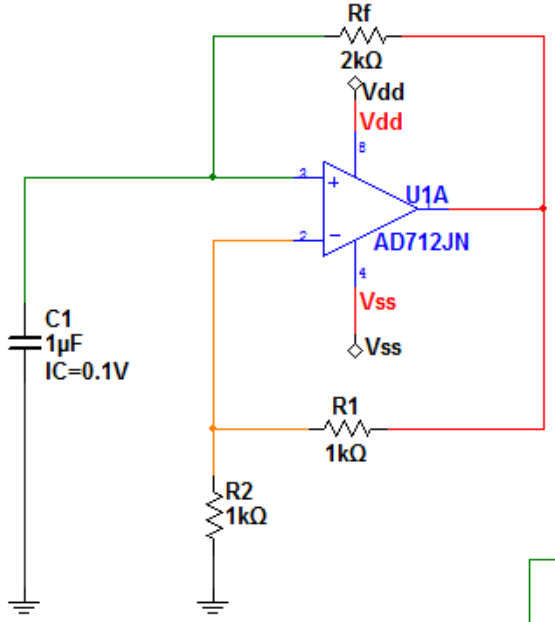
Analyze and simulate circuit [4] shown below. Specifically, sketch Vo(t) in the circuit below. Note: You should realize circuit physically on a breadboard.

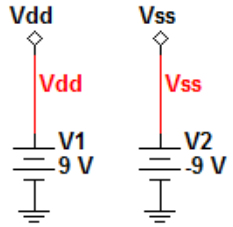


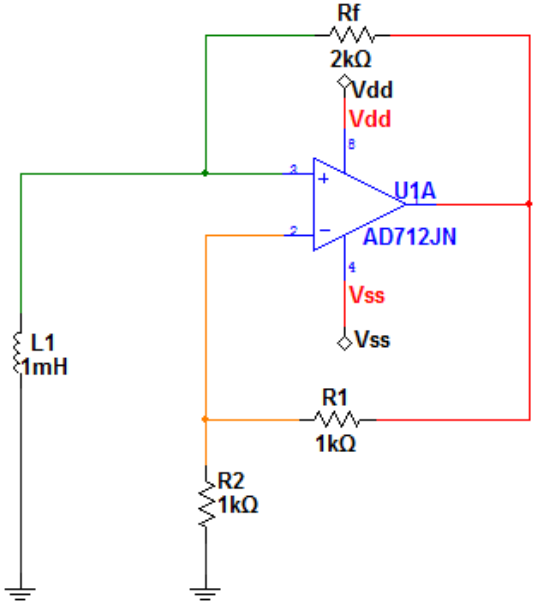
NOTE: When you have questions, please STOP ME and ASK!

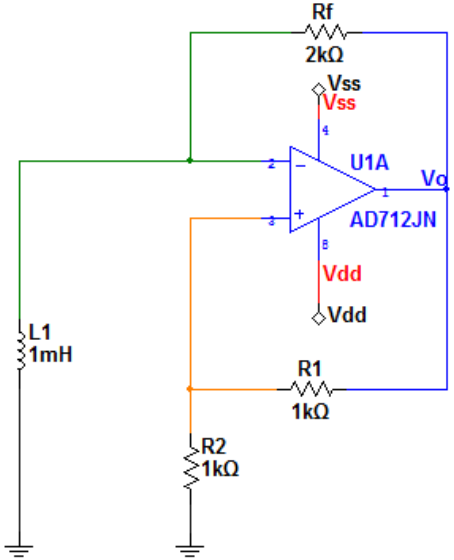
If you have seen this circuit before…

What does each circuit below do (answer in ONE word)?



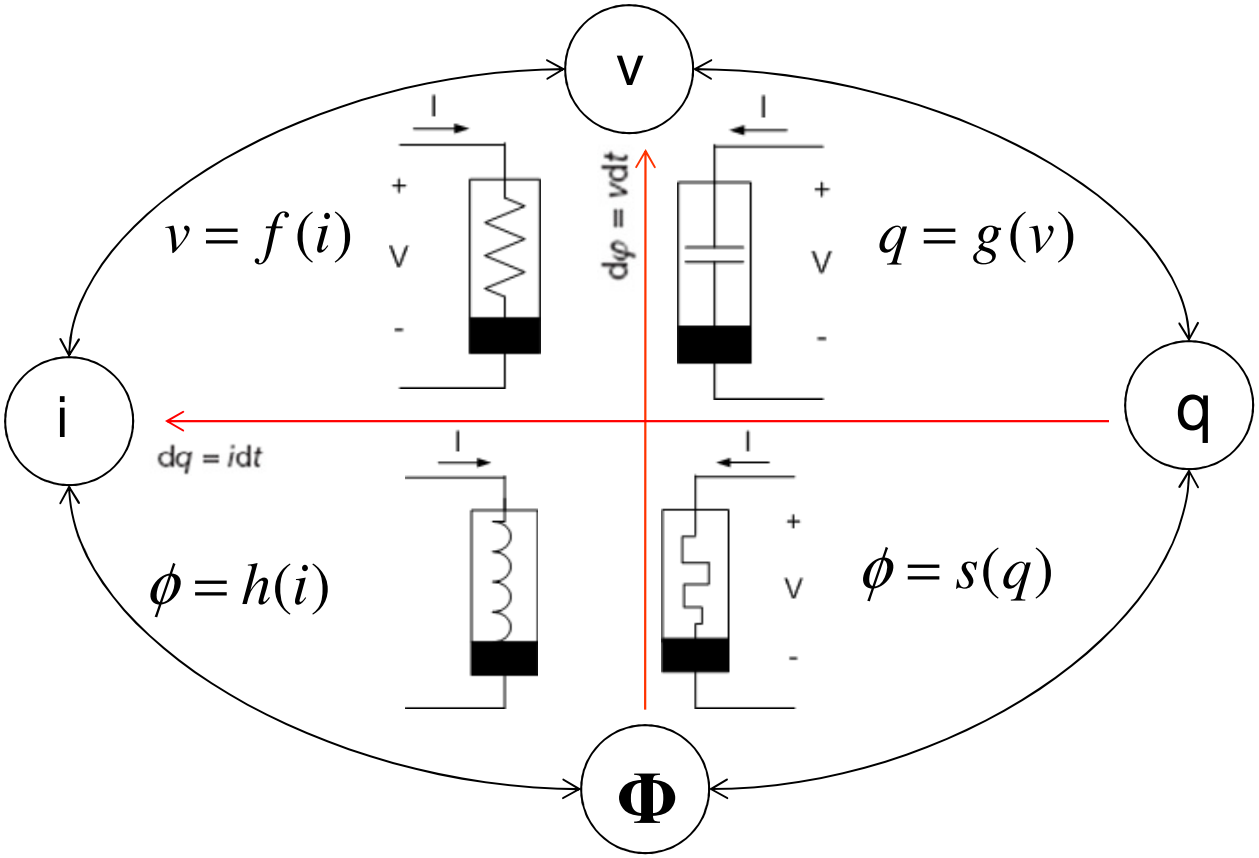






Wait till end of lecture ☺ …

Introduction : Fundamental Circuit Theory [1-3]

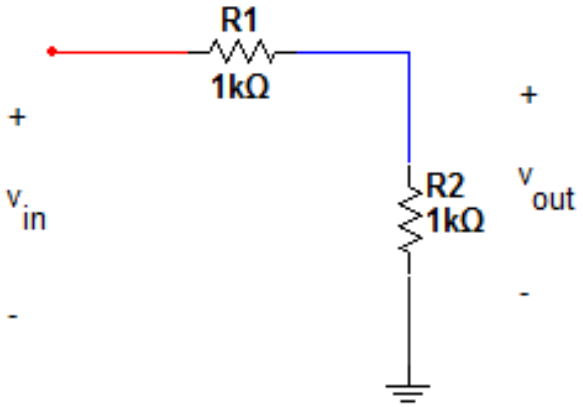


Memristors were first postulated by Leon. O Chua in 1971 [1]

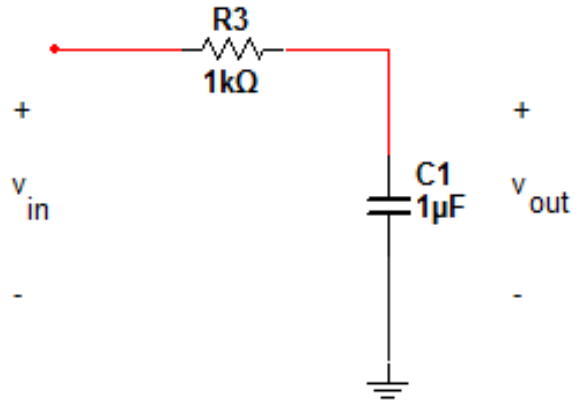
Introduction : Static vs. Dynamical Systems

1. Mathematical definition of a system

$$y(t) = S(x)(t) \quad y, x : \mathbb{R} \to \mathbb{R}, t \in \mathbb{R} \qquad (1)$$



$$v_{out}(t) = \frac{R2}{R1+R2} v_{in}(t) \qquad (2)$$



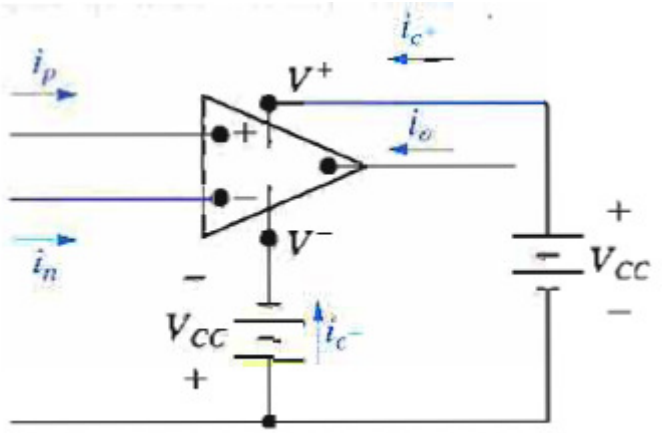
$$R3C1\frac{dv_{out}}{dt} + v_{out} = v_{in} \qquad (3)$$

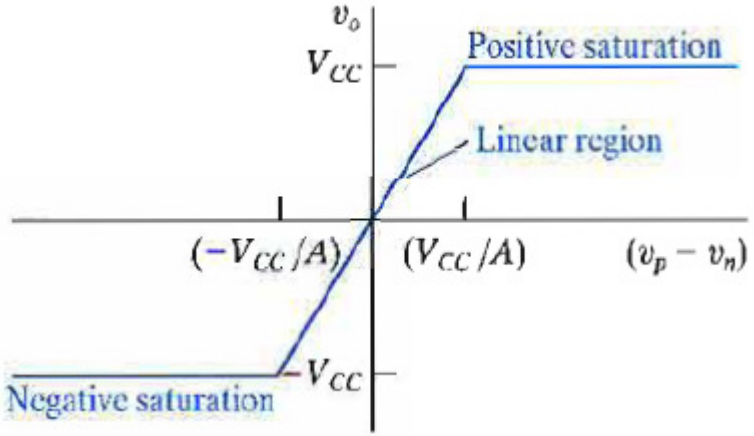
$$v_{out}(t) = v_{out}(0)e^{-t/(R3C1)} + \frac{1}{R3C1}\int_0^t e^{\frac{-(t-\tau)}{R3C1}} v_{in}(\tau)d\tau \qquad (4)$$

2. Concept of a linear time-invariant system

3. Various system behaviors: stable, unstable

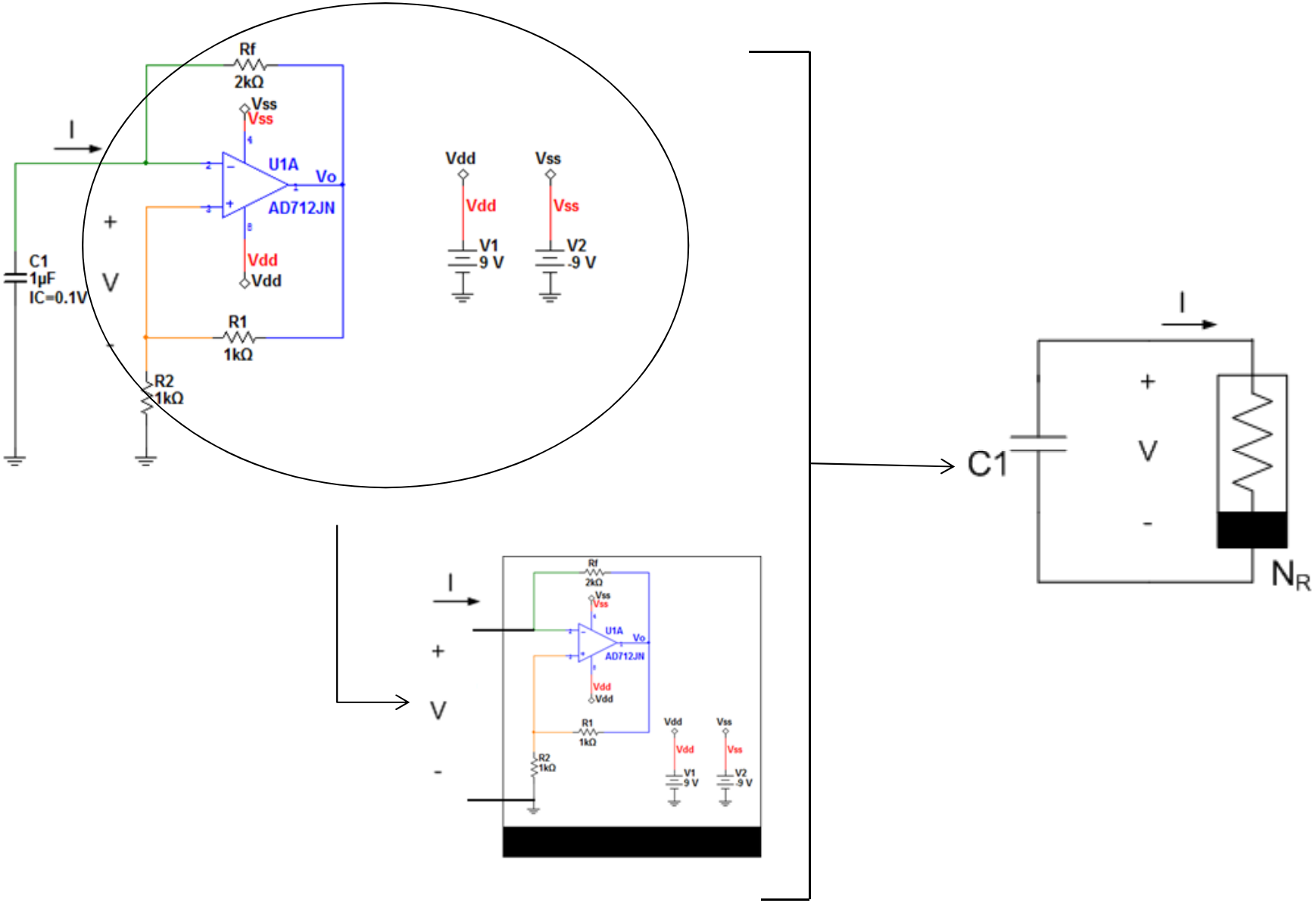
“Review” : The Operational Amplifier [5]





$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC}, \\ A(v_p - v_n) & -V_{CC} \leq A(v_p - v_n) \leq +V_{CC}, \\ +V_{CC} & A(v_p - v_n) > +V_{CC}. \end{cases} \quad (5)$$

Step 1: Understand Problem



Step 2: Devise a Plan

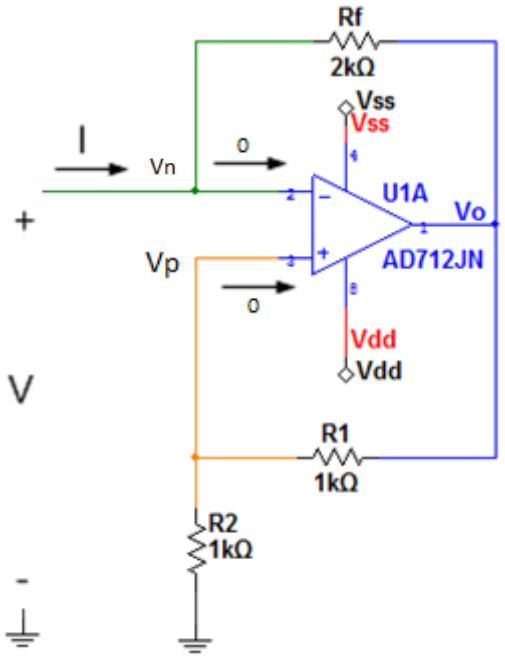
1. Derive i-v graph of $N_R = I(V)$ using op-amp ideas from lecture 1

2. Use $I = -C\dfrac{dV}{dt} \quad (6)$ (concepts from circuit theory)

3. Use (1) to find and sketch Vo(t)

Step 3 (a): Carry out the plan

1. Terminology



2. Circuit Equations

$$I = \frac{v_n - v_0}{R_f}$$

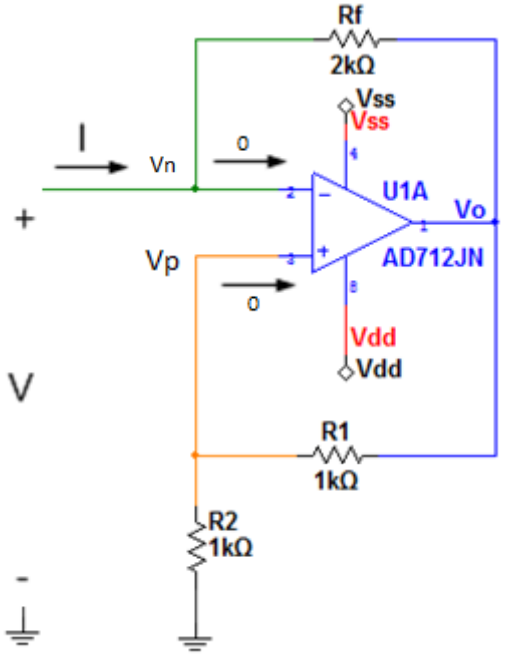
$$= \frac{V - v_0}{R_f} \qquad (7)$$

$$v_p = \frac{R_2}{R_1 + R_2} v_0 \qquad (8)$$

3. Recall three modes of operation for an op-amp : linear region, positive saturation and negative saturation.

Step 3 (b): Carry out the plan

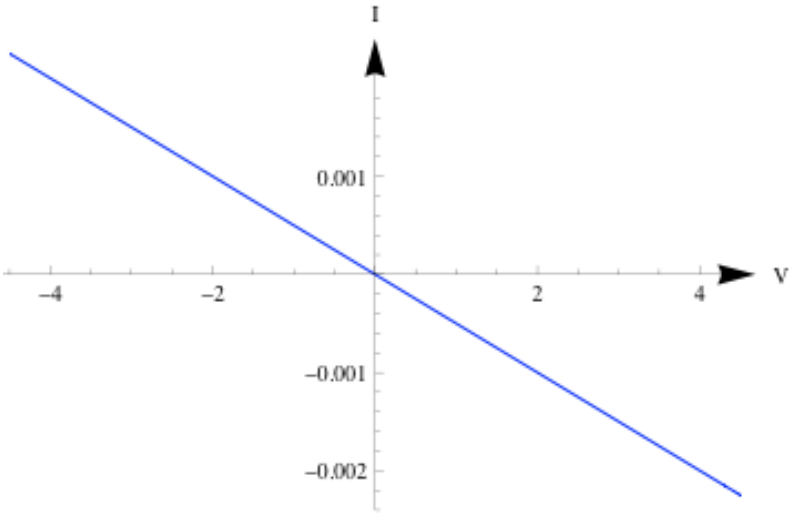
1. Op-amp in linear region



2. Circuit Equations

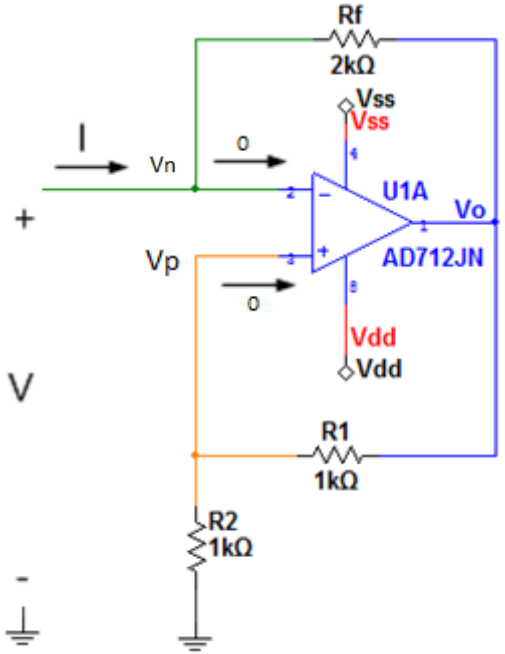
$$I = -\frac{V}{R_f} \qquad (9)$$

3. I-V graph



Step 3 (c): Carry out the plan

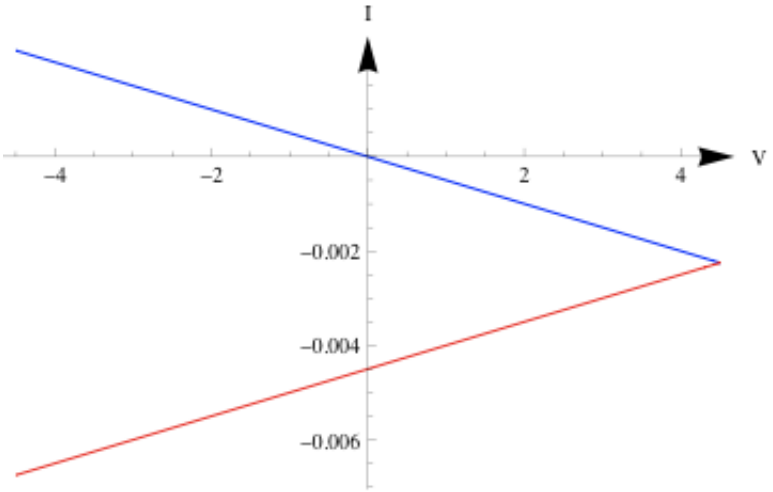
1. Op-amp in positive saturation



2. Circuit Equations

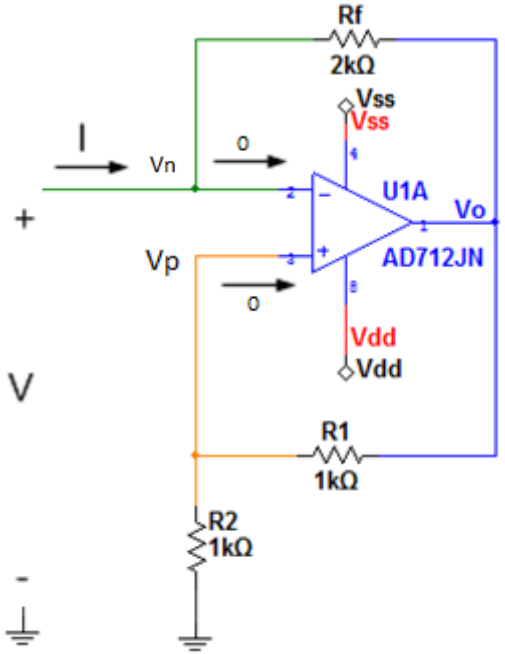
$$I = \frac{V - 9}{R_f} \quad \text{V} < 4.5 \text{ volts} \quad (10)$$

3. I-V graph



Step 3 (d): Carry out the plan

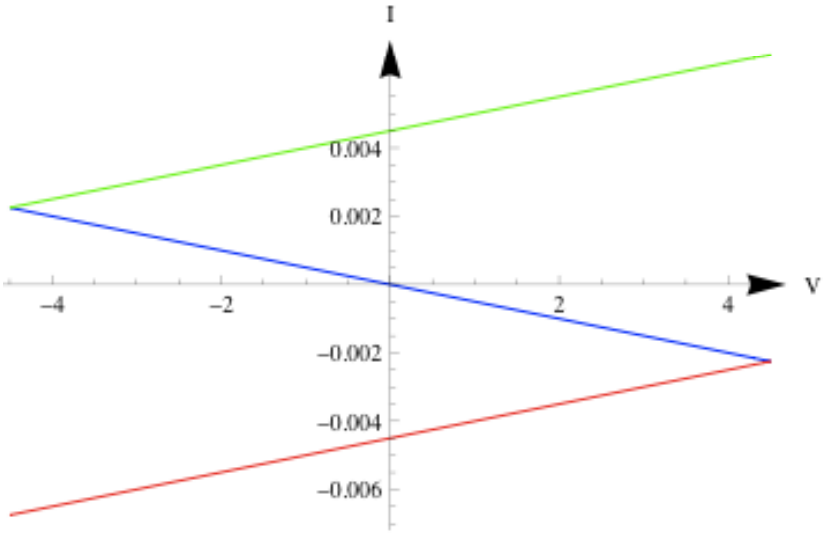
1. Op-amp in negative saturation



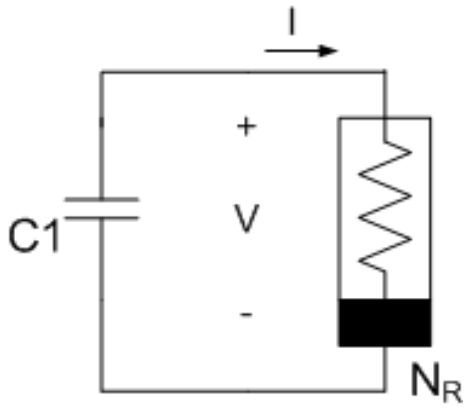
2. Circuit Equations

$$I = \frac{V + 9}{R_f} \quad \text{V} > \text{-4.5 volts} \quad (11)$$

3. I-V graph



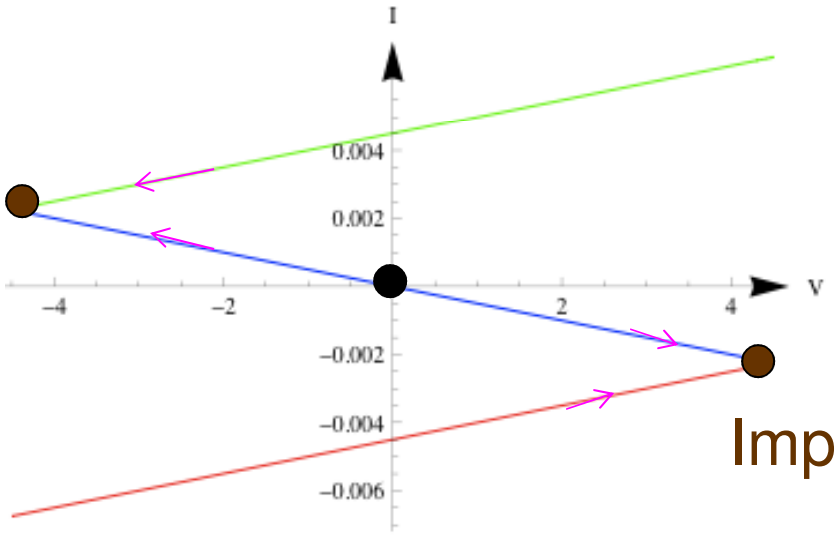
Step 3(e) : Finding Vo(t)



$$I = -C\frac{dV}{dt} \quad (12)$$

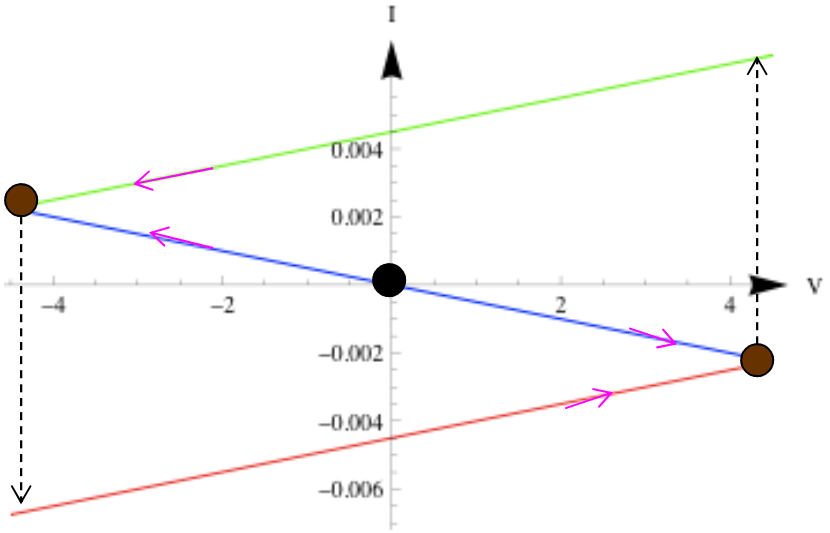
$$\text{Equilibrium points} : \frac{dV}{dt} \triangleq 0 \quad (13)$$

$$\text{Dynamic Route} \triangleq \begin{cases} I > 0 \quad \dot{V} < 0 \quad (C > 0) \\ I < 0 \quad \dot{V} > 0 \quad (C > 0) \end{cases} \quad (14)$$

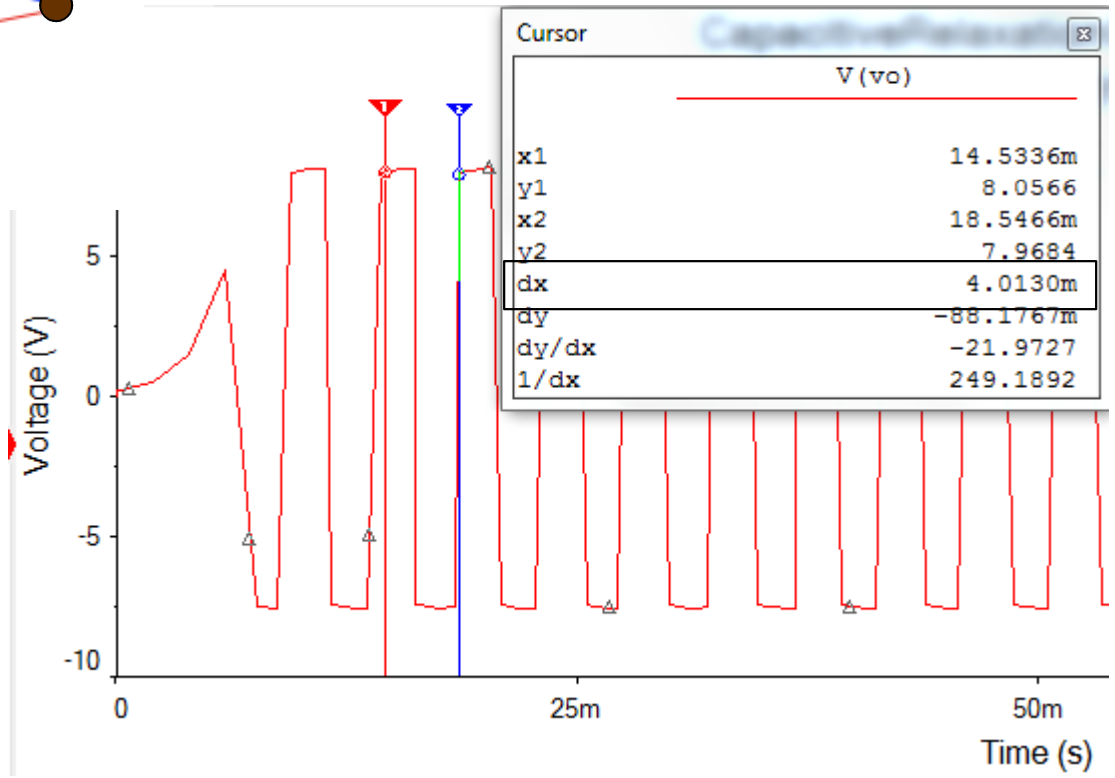


Impasse Points

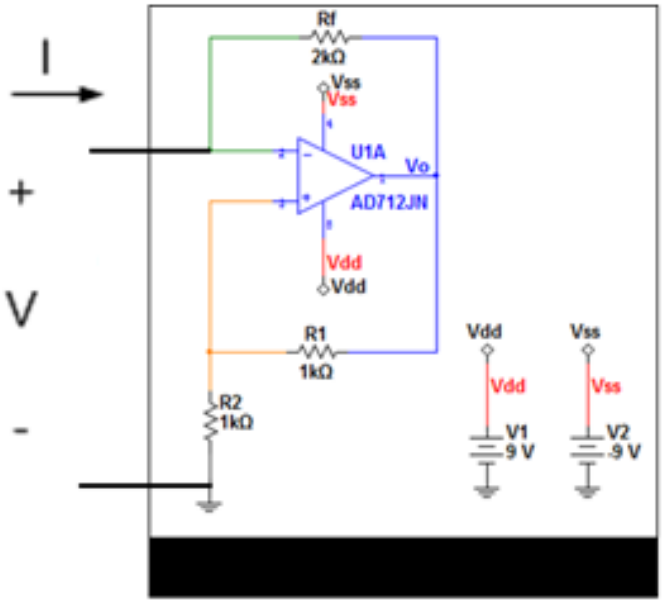
Step 3(e) : Sketching Vo(t)

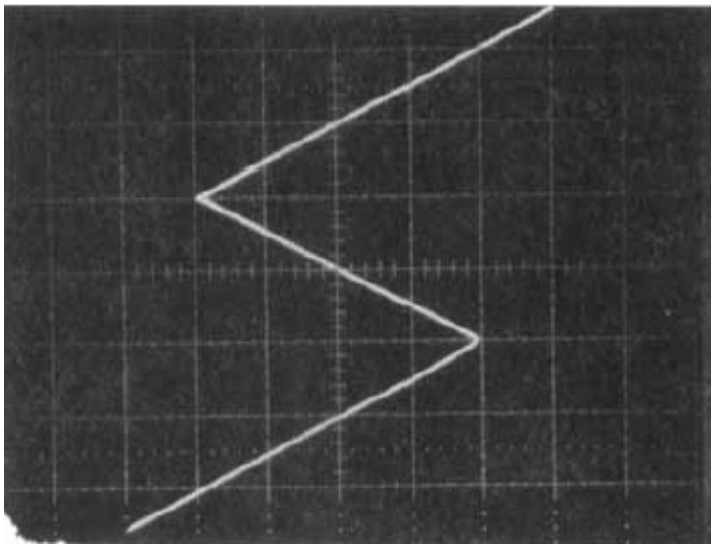


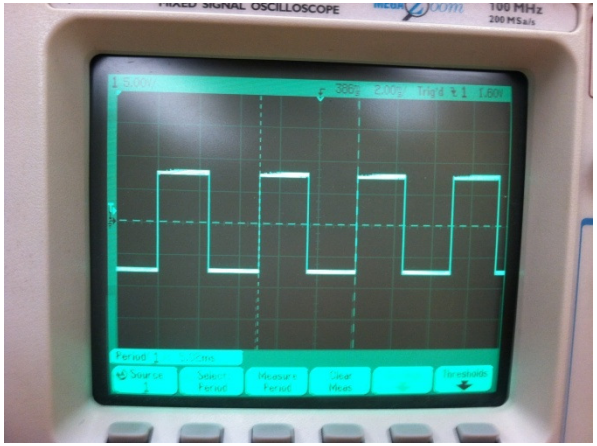
Assignment: Compute oscillation period!



Step 4: Check your answer

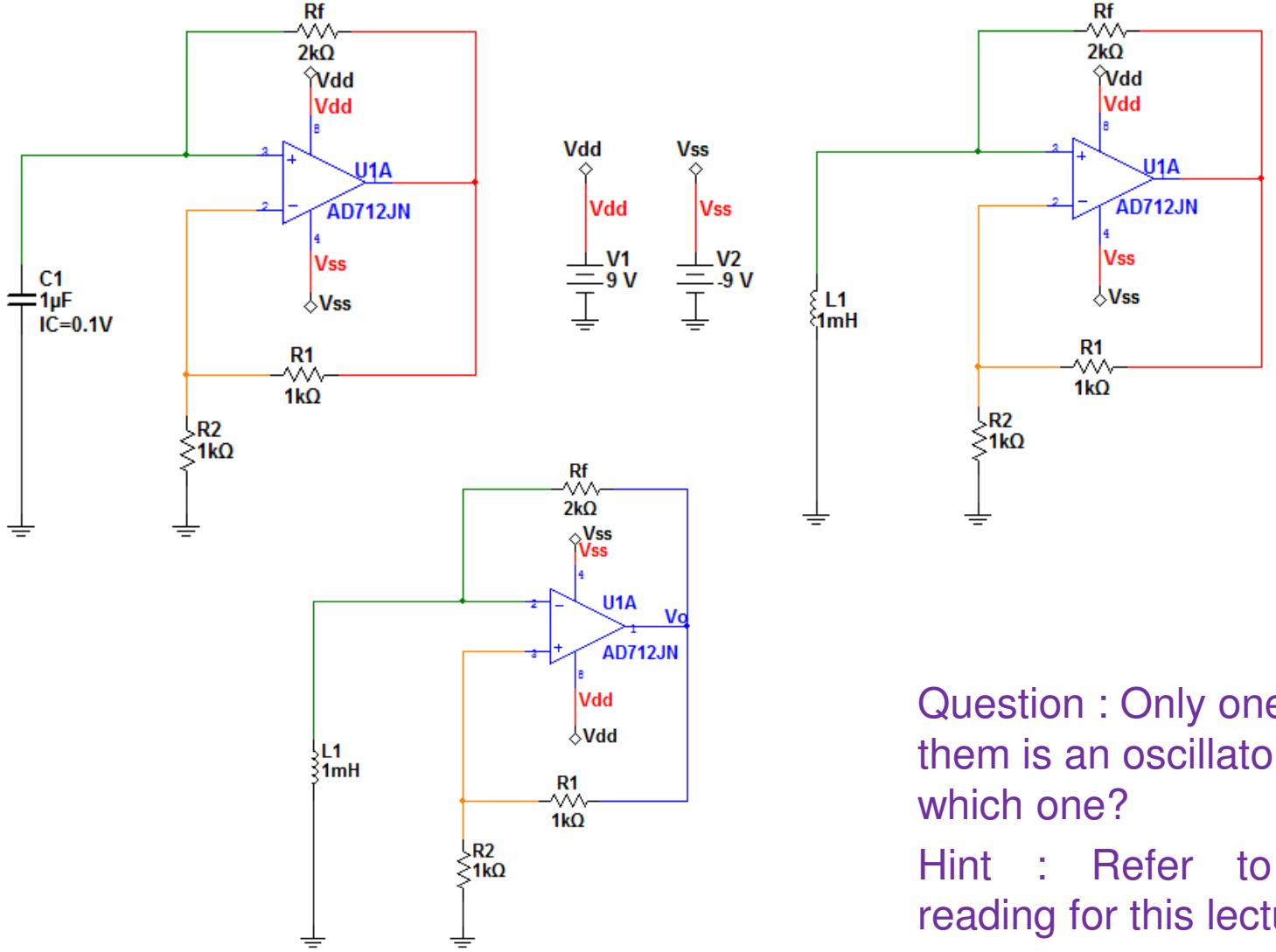






Remember us ☺?

What does each circuit below do (answer in ONE word)?



Question : Only one of them is an oscillator, which one?

Hint : Refer to the reading for this lecture!

References

1. Chua, L. O. “Memristor – The Missing Circuit Element”. *IEEE Transactions on Circuit Theory*, Vol. CT-18, No. 5, pp. 507- 519. September 1971.
2. Chua, L. O. and Kang, S. M. “Memristive Devices and Systems”. *Proceedings of the IEEE*, Vol. 64, No. 2, pp. 209- 223. February 1976.
3. Chua, L. O., Desoer, C. A. and Kuh, E. H. “Linear and Nonlinear Circuits”. McGraw-Hill, 1987.
4. Kennedy, M. P. and Chua, L. O. “Hysteresis in Electronic Circuits : A Circuit Theorist’s Perspective”. International Journal of Circuit Theory and Applications, Vol. 19, pp. 471 – 515, 1991.
5. Nilsson, J. W. and Riedel, S. “Electric Circuits”. 9th Edition, Pearson, 2011.