How to Think and Learn: An Example Using Maxwell's Equations

(title inspired by "How to Solve It", Polya, G.)

Talk given at Vidhya Mandir, Mylapore, Chennai, India

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Many thanks to my colleagues from MSOE: Dr. Jevtic and Dr. Thomas for inspiring me to do the work I do!

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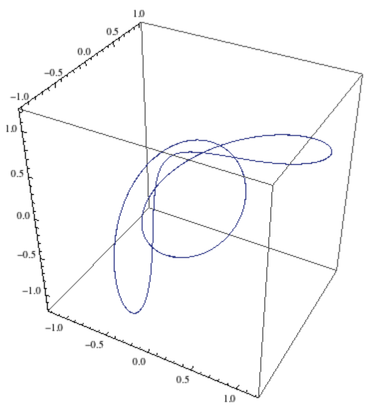
What *do* I work on?

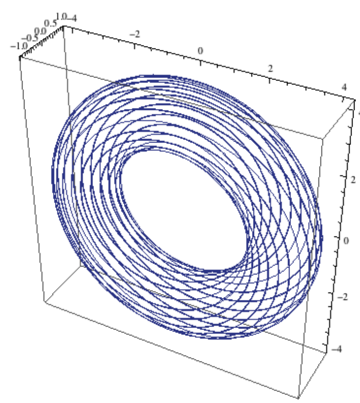
Nonlinear Dynamical Systems and Embedded Systems

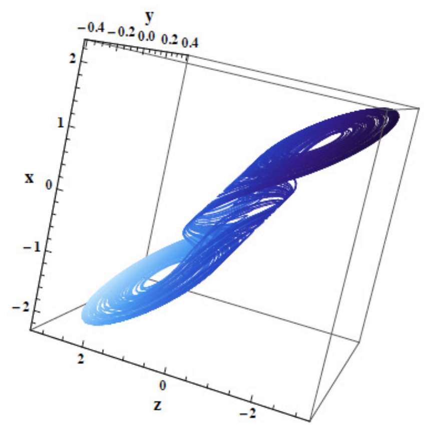
- Applications and Mathematical properties of the Muthuswamy-Chua system
  - Potential Applications to Turbulence Modeling
    (MSOE; University of Western Australia, Perth, Australia)
  - Chaotic Hierarchy and Flow Manifolds
    (MSOE; University of Western Australia, Perth, Australia; I.U.T. de Toulon, La Garde Cedex, France)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (FPGAs)
  (MSOE; Vellore Institute of Technology; University Putra Malaysia, Malaysia; Springer-Verlag)
- Pattern Recognition Using Cellular Neural Networks on FPGAs
  (MSOE; Altera Corporiation)
- Practical Memristors: discharge tubes, PN junctions and Josephson Junctions
  (MSOE; IIT Chennai; University of Western Australia, Perth, Australia;
  University of California, Berkeley; Vellore Institute of Technology, Vellore, India)

Education

- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)







Outline

I. Prerequisites for understanding this talk:
   1. *First course in vector algebra: dot product, cross product, divergence, curl
   2. Willingness to think and learn

II. A Brief History of Maxwell's Equations
   1. Michael Faraday (and others)
   2. James Clerk Maxwell
   3. Gibbs, Heaviside etc.

III. Thinking AND Learning about Maxwell's Equations

IV. Problem Solution

V. Conclusion

VI. References

A Brief History of Maxwell's Equations: Michael Faraday (and others)

Faraday's name has been rightfully permanently linked with electricity and magnetism. He was 30 (1821) when he discovered electromagnetic rotations and 40 (1831) when he discovered induction, using it to produce the first electric generator and transformer [6].

A Brief History of Maxwell's Equations: James Clerk Maxwell

From Maxwell's treatise on electromagnetism [8]:

The theory I propose may therefore be called a theory of the *Electromagnetic Field*,

(p. 460)

The conception of the propagation of transverse magnetic disturbances to the exclusion of normal ones is distinctly set forth by Professor FARADAY* in his "Thoughts on Ray Vibrations." The electromagnetic theory of light, as proposed by him, is the same in substance as that which I have begun to develope in this paper, except that in 1846 there were no data to calculate the velocity of propagation.

(p. 466)

For Electromagnetic Momentum	F	G	H
„ Magnetic Intensity	α	β	γ
„ Electromotive Force	P	Q	R
„ Current due to true conduction	p	q	r
„ Electric Displacement	f	g	h
„ Total Current (including variation of displacement)	p'	q'	r'
„ Quantity of free Electricity	e		
„ Electric Potential	Ψ		

Between these twenty quantities we have found twenty equations, viz.

Three equations of Magnetic Force	(B)
„ Electric Currents	(C)
„ Electromotive Force	(D)
„ Electric Elasticity	(E)
„ Electric Resistance	(F)
„ Total Currents	(A)
One equation of Free Electricity	(G)
„ Continuity	(H)

(p. 486)

A Brief History of Maxwell's Equations: Gibbs, Heaviside etc.

Maxwell's Equations reformulated using vector notation [3]:

$$\vec{\nabla} \cdot \vec{D} = \rho, \oint \vec{E} \cdot d\vec{S} = \frac{q_{encl}}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

Maxwell's Equations reformulated using geometric algebra [1]:

$$\Box F = J$$

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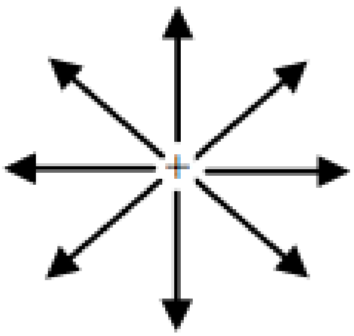
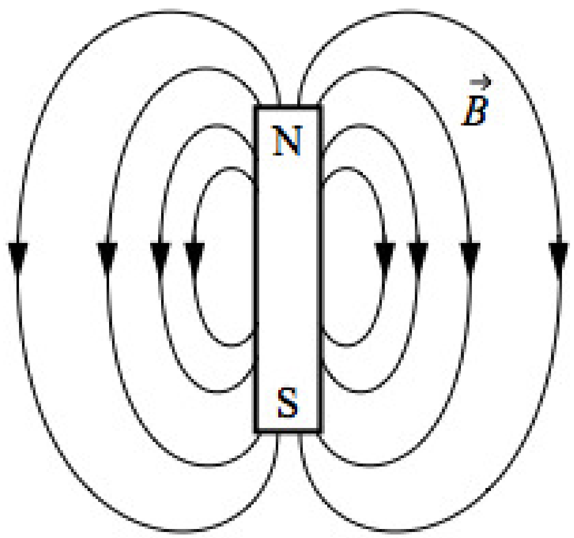
Thinking about Maxwell's Equations [2]

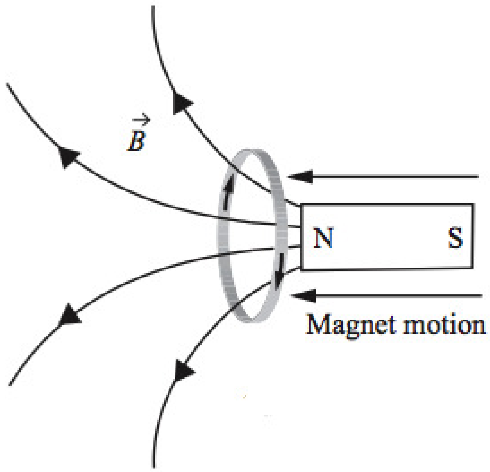


$$\longleftrightarrow \quad \vec{\nabla} \cdot \vec{D} = \rho \quad \longleftrightarrow \quad \oint \vec{E} \cdot \hat{n} \; da = \frac{q_{encl}}{\varepsilon_0}$$

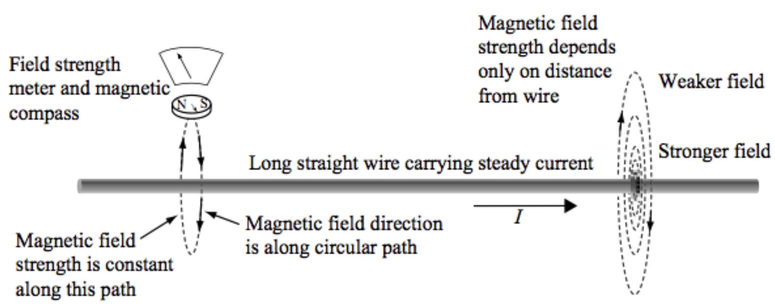


$$\longleftrightarrow \quad \vec{\nabla} \cdot \vec{B} = 0$$

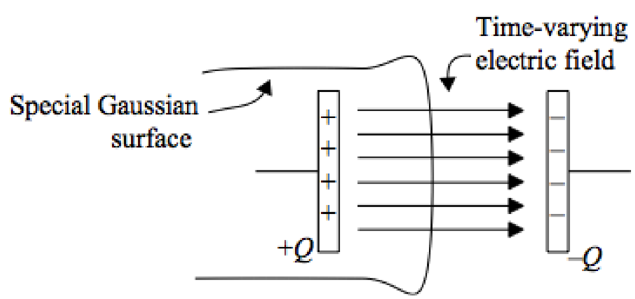
Thinking about Maxwell’s Equations (contd.) [2]



$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

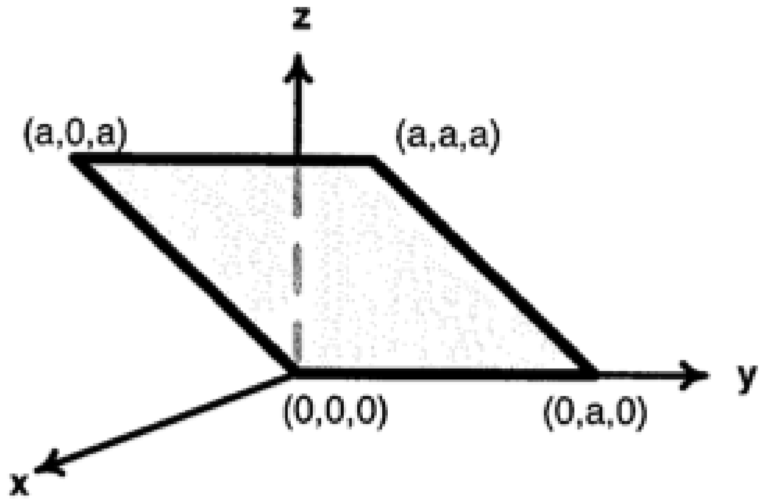


$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \frac{\partial \vec{D}}{\partial t}$$



Learning about Maxwell's Equations - Example [4]

Consider an electric field $\vec{E} = E_0\ \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

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Solution

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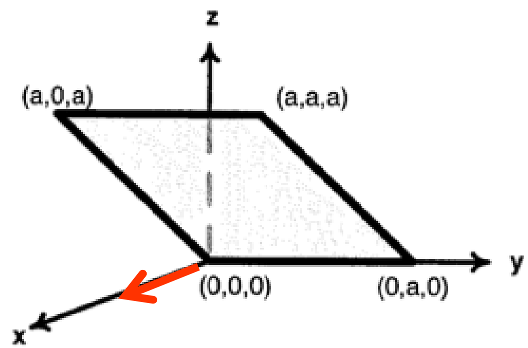

$$\Phi_E = \oint \vec{E} \cdot \hat{n}\ da$$

$$= \oint \left|\vec{E}\right| \left|\hat{n}\right| \cos(0^\circ) da$$

$$= E_0 \oint da$$

$$= E_0 \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & a & 0 \\ a & 0 & a \end{matrix} \right\|$$

$$= E_0 a^2 \text{ V.m}$$

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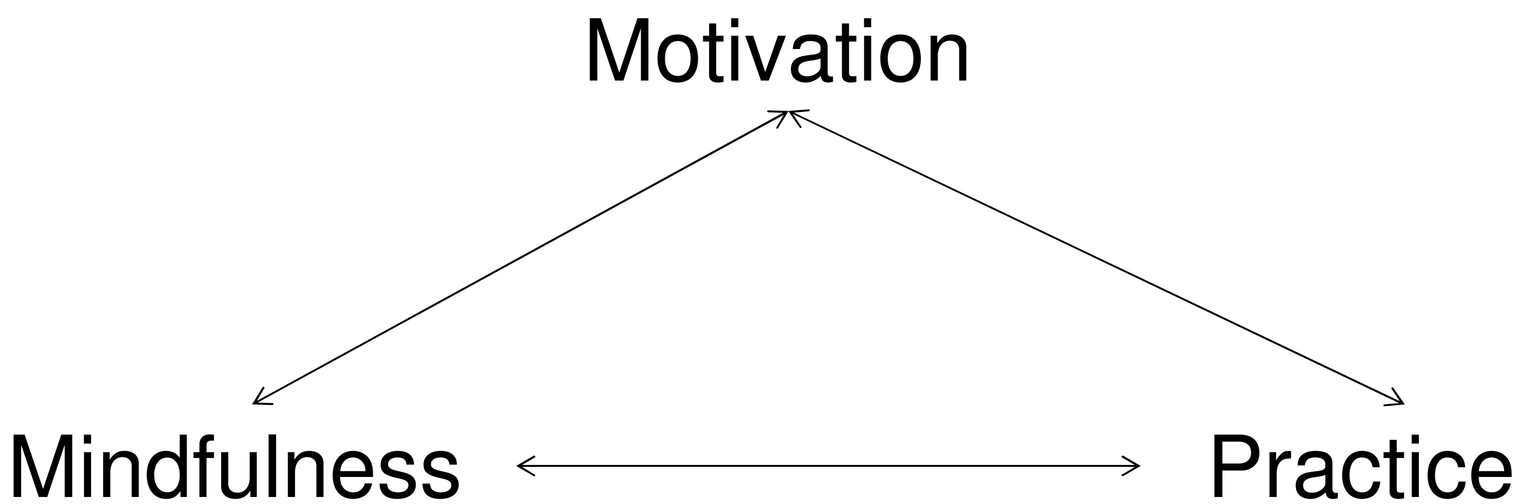

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1. Doran, C.J.L. and Lasenby, A. N. *Geometric Algebra for Physicists*. Cambridge University Press, 2003.
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4. *IIT JEE 2011 Paper I – Physics, Question 26.*
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7. Konnikova, M. *Mastermind: How to Think Like Sherlock Holmes.* Pengiun, 2013.
8. Maxwell, J.C. *A Dynamical Theory of the Electromagnetic Field.* Philosophical Transactions of the Royal Society of London. pp. 459 - 512, 1865.

Why study at a primarily undergraduate school (MSOE) in the US of A?