Chaotic Dynamics of the Muthuswamy-Chua[1] System

IIT Chennai

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MSOE

[1]“On the Integrability of a Muthuswamy-Chua System”, Llibre, J. and Valls, C. Journal of Nonlinear Mathematical Physics, Vol. 19, No. 4, pp. 1250029 – 1250041 December 2012.

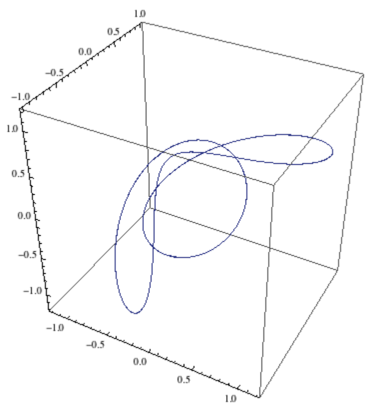
What do I work on?

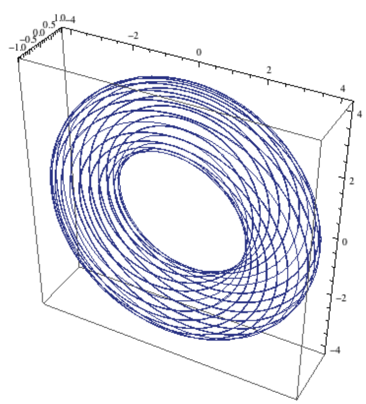
Nonlinear Dynamical Systems and Embedded Systems

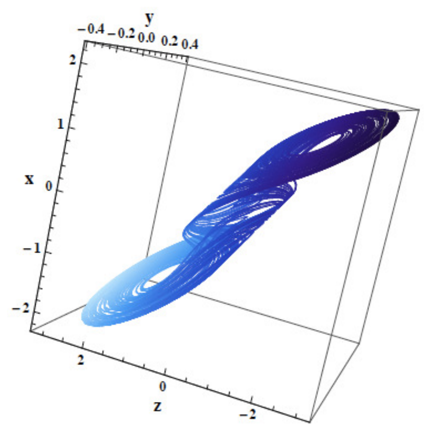
- Applications and Mathematical properties of the Muthuswamy-Chua system
    - Potential Applications to Turbulence Modeling
      (MSOE; University of Western Australia, Perth, Australia)
    - Chaotic Hierarchy and Flow Manifolds
      (MSOE; University of Western Australia, Perth, Australia; I.U.T. de Toulon, La Garde Cedex, France)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (FPGAs)
  (MSOE; Vellore Institute of Technology; University Putra Malaysia, Malaysia; Springer-Verlag)
- Pattern Recognition Using Cellular Neural Networks on FPGAs
  (MSOE; Altera Corporiation)
- Practical Memristors: discharge tubes, PN junctions and Josephson Junctions
  (MSOE; IIT Chennai; University of Western Australia, Perth, Australia;
  University of California, Berkeley; Vellore Institute of Technology, Vellore, India)

Education

- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)

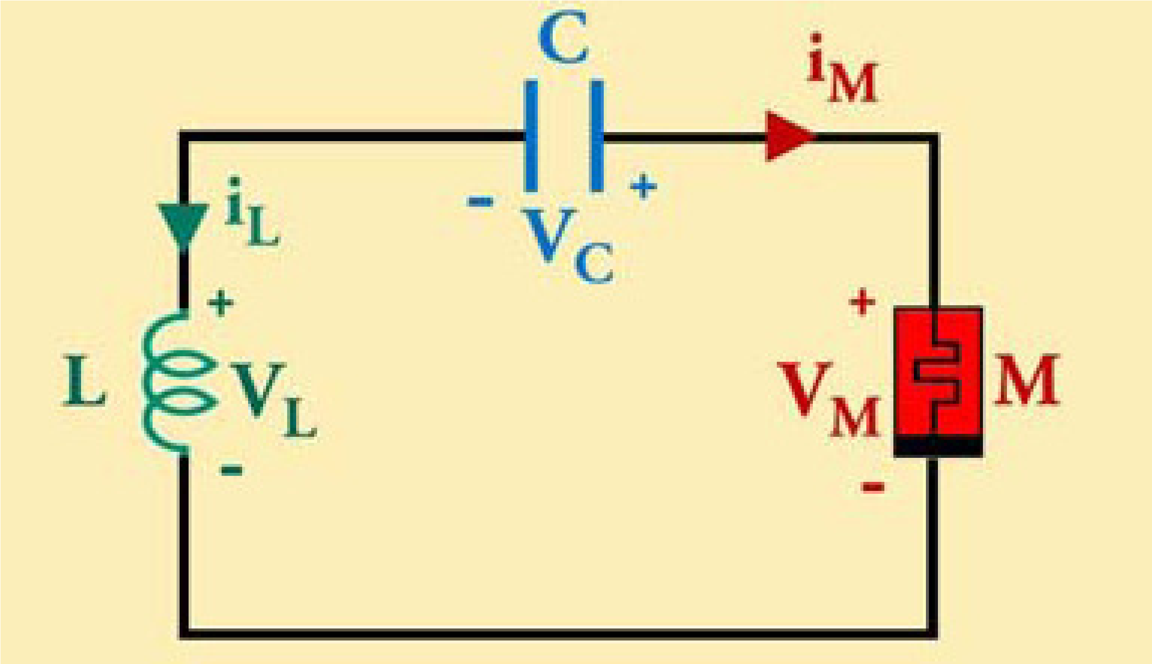






Goal of This Talk

Obtain chaos in the circuit [5] below:



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III. The Memristor

1. Properties of the memristor
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Introduction to chaos

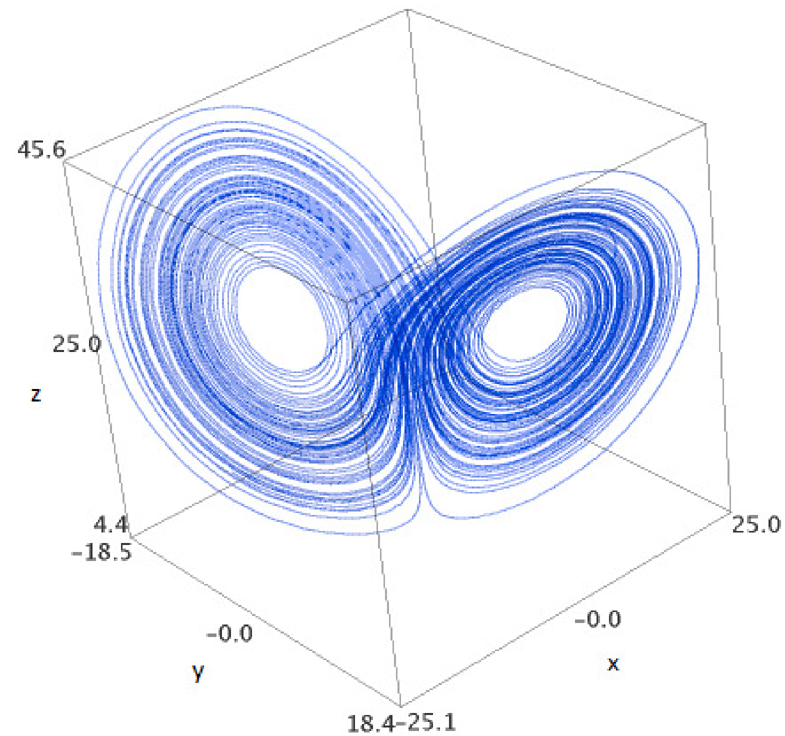
- "Birth" of Chaos: Lorenz Attractor [6]
  - Edward Lorenz introduced the following nonlinear system of differential equations as a crude model of weather in 1963:

$$\begin{aligned}\dot{x} &= -\sigma\cdot x+\sigma\cdot y\\ \dot{y} &= \rho\cdot x-y-x\cdot z \qquad (1)\\ \dot{z} &= -\beta\cdot z+x\cdot y\end{aligned}$$

Parameters: $\sigma=10, \rho=28, \beta=\frac{8}{3}$

ICs: $x_0=10, y_0=20, z_0=30,$

Simulation time: 100 seconds



  - Lorenz discovered that model dynamics were extremely sensitive to initial conditions and the trajectories were aperiodic but bounded.
  - But, does chaos exist *physically?* Answer is: YES. For example, chaotic circuits by Sprott [7].

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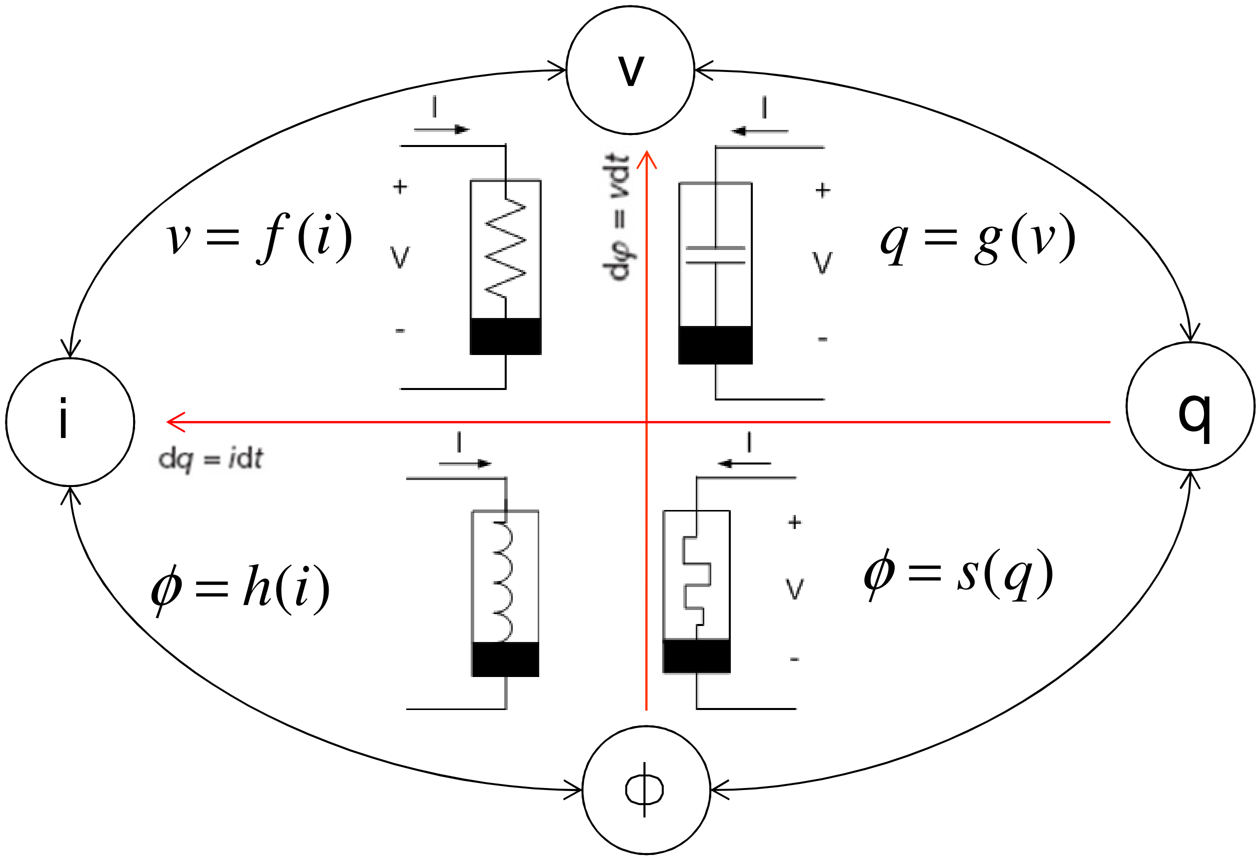
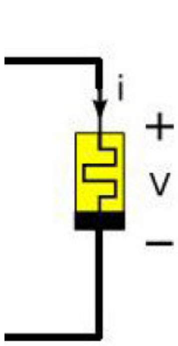
The Fundamental Circuit Elements



Memristors were first postulated by Leon. O Chua in 1971 [2]

Properties of the Memristor [2]

Circuit symbol:



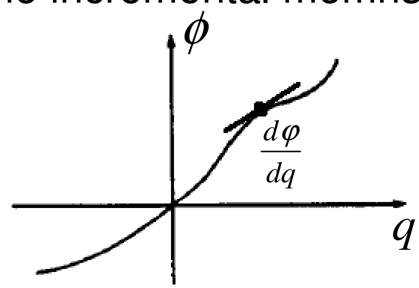
A memristor defines a *relation* of the from: $g(\phi, q) = 0 \quad (2)$

If g is a single-valued function of charge (flux), then the memristor is charge-controlled (flux-controlled)

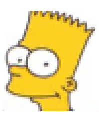
Memristor i-v relationship:

M(q(t)) is the incremental memristance

$$v(t) \triangleq \frac{d\phi}{dt} = \frac{d\phi}{dq}\frac{dq}{dt} \triangleq M(q(t))i(t) \qquad (3)$$

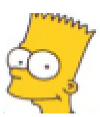


Q1: Why is the memristor called “memory resistor”?



Because of the definition of memristance: $v(t) = M(q(t))i(t) = M\left(\int_{-\infty}^{t} i(\tau)\right)i(t)$

Q2: Why is the memristor not relevant in linear circuit theory?



1. If M(q(t)) is a constant: $v(t) = M(q(t))i(t) = Mi(t) = Ri(t)$
2. Principle of superposition is not* applicable:

$$M\left(\int_{-\infty}^{t}(i_1+i_2)(\tau)\right)(i_1+i_2)(t) = M\left(\int_{-\infty}^{t}(i_1)(\tau) + \int_{-\infty}^{t}(i_1)(\tau)\right)(i_1+i_2)(t) \neq M\left(\int_{-\infty}^{t}(i_1)(\tau)\right)i_1(t) + M\left(\int_{-\infty}^{t}(i_2)(\tau)\right)i_2(t)$$

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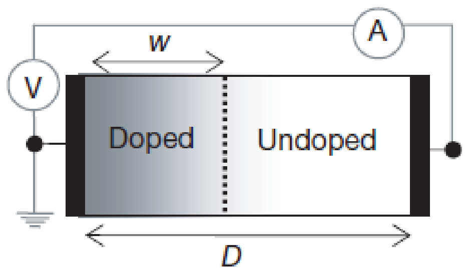
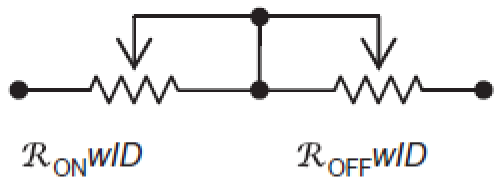
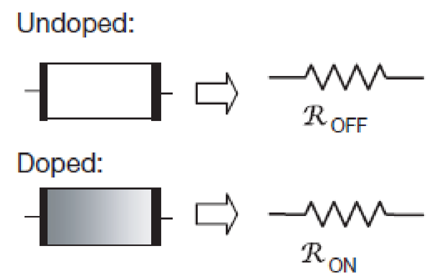
Hewlett-Packard's memristor [9]

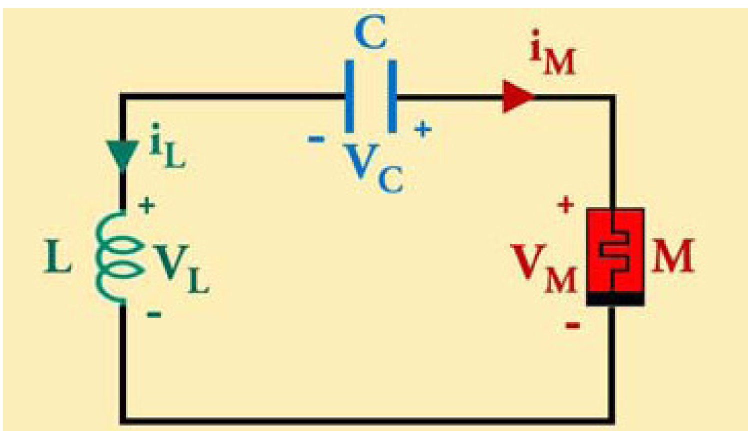






$$v(t) = \left(\mathcal{R}_{ON}\frac{w(t)}{D} + \mathcal{R}_{OFF}\left(1 - \frac{w(t)}{D}\right)\right) i(t) \xrightarrow[\mathcal{R}_{ON} \ll \mathcal{R}_{OFF}]{} M(q) = \mathcal{R}_{OFF}\left(1 - \frac{\mu_V \mathcal{R}_{ON}}{D^2} q(t)\right)$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \mu_V \frac{\mathcal{R}_{ON}}{D} i(t)$$



Circuit equations:

System equations:

$$\dot{v}_c = \frac{i_L}{C}$$

$$\dot{q}_M = -i_L \xrightarrow{x \triangleq v_c,\, z \triangleq q_M,\, y \triangleq i_L}$$

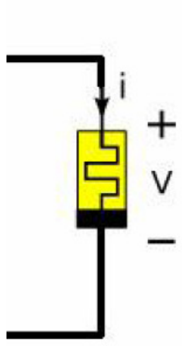
$$\dot{x} = \frac{y}{C}$$

$$\dot{z} = -y \qquad (5)$$

$$\dot{y} = \frac{-1}{L}\left(x + M(z)y\right)$$

$$v_L + v_C = v_M \Rightarrow L\frac{di_L}{dt} = -v_C + M(q_M)i_M \Rightarrow i'_L = \frac{-1}{L}\left(v_C + M(q_M)i_L\right)$$

Memristive Devices [3]



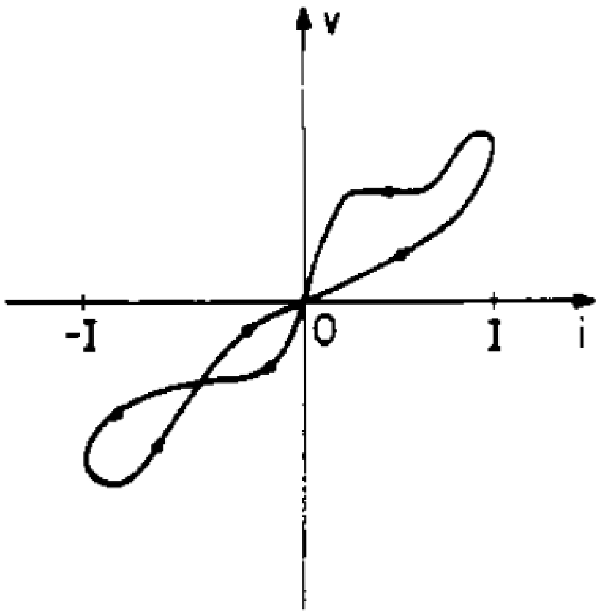
$$v \triangleq R(z,i)i$$
$$\dot{z} = f(z,i) \qquad (6)$$

$$\xrightarrow{z \triangleq q,\, R(z,i) \triangleq M(q)}$$

$$v \triangleq M(q)i$$
$$\dot{q} = i$$

The functions *R* and *f* are defined as:

$$R : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$
$$f : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}^1$$



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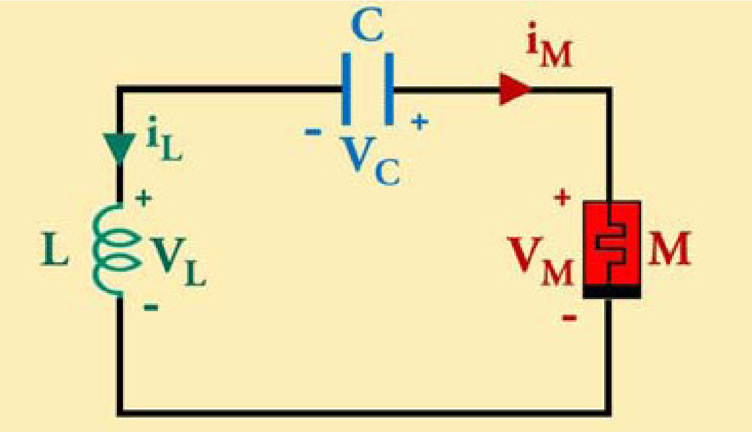
Derivation of Circuit Equations [5]

$$v_M \triangleq R(z, i_M) i_M$$

$$\dot{z} = f(z, i_M)$$



Circuit equations:

$$\dot{v}_c = \frac{i_L}{C}$$

$$i'_L = \frac{-1}{L}\left(v_C + R(z, i_L) i_L\right)$$

$$\dot{z} \triangleq f(z, i_L)$$

$$\xrightarrow{x \triangleq v_c,\; y \triangleq i_L}$$

System equations:

$$\dot{x} = \frac{y}{C}$$

$$\dot{y} = \frac{-1}{L}\left(x + R(z, y) y\right) \quad (7)$$

$$\dot{z} = f(z, y)$$

Specifically:

$$\dot{x} = \frac{y}{C}$$

$$\dot{y} = \frac{-1}{L}\left(x + \beta(z^2 - 1) y\right) \quad (8)$$

$$\dot{z} = -y - \alpha z + yz$$

Parameters:

$$\xrightarrow{}$$

$$C = 1, L = 3$$

$$\beta = \frac{3}{2}, \alpha = \frac{3}{5}$$

$$\dot{x} = y$$

$$\dot{y} = \frac{-x}{3} - \frac{z^2 y}{2} + \frac{y}{2} \quad (9)$$

$$\dot{z} = -y - 0.6z + yz$$

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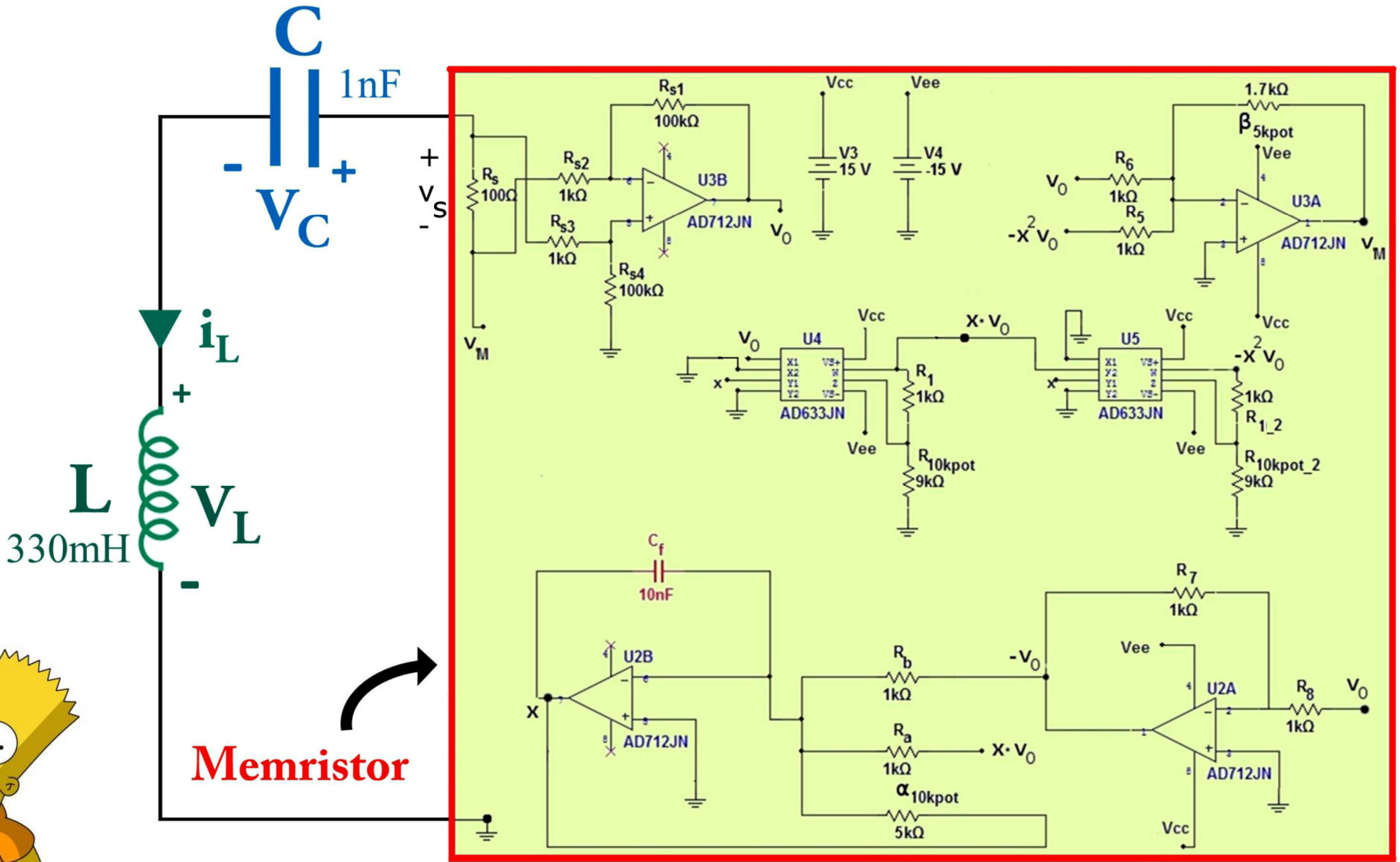
Physical Realization of the Memristor [5]



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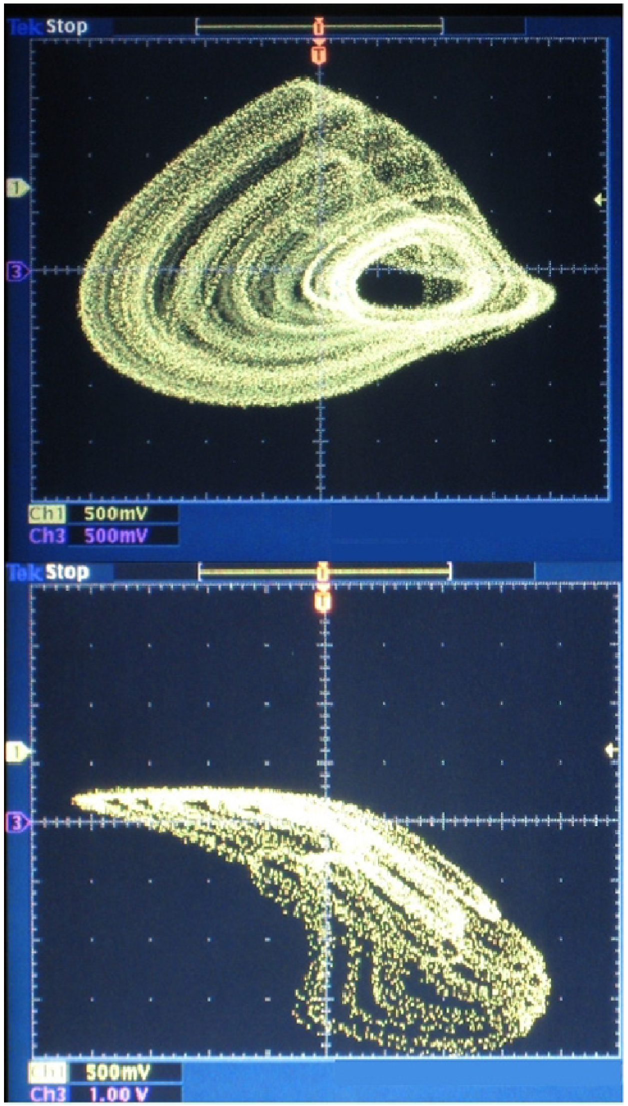
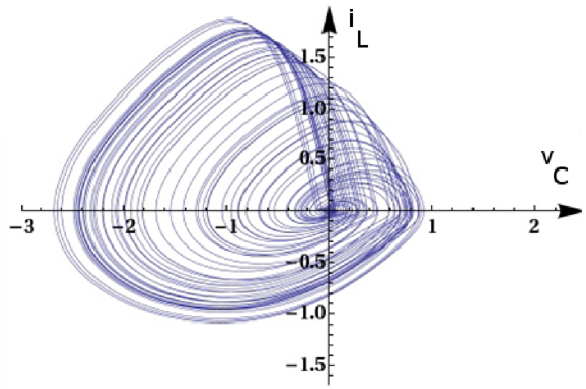
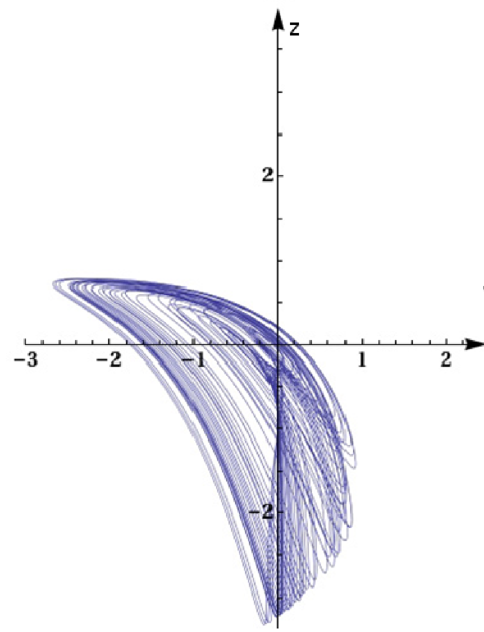
Attractors from the Circuit [5]







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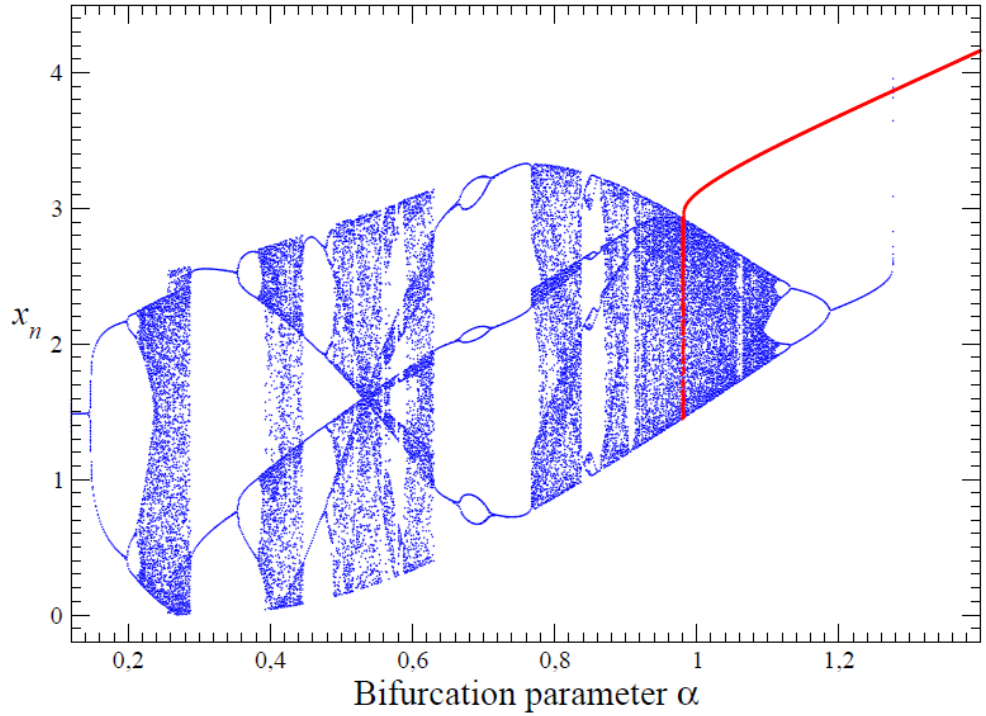
Rigorous Mathematical Analysis

Paper by Ginoux et. al. "Topological Analysis of Chaotic Solution of Three-Element Memristive Circuit". International Journal of Bifurcation and Chaos, Vol. 20, No. 11., pp. 3819 – 3829, Nov. 2010.

$$\dot{x} = y$$

$$\dot{y} = -\frac{x}{3} + \frac{y}{2} - \frac{yz^2}{2}$$

$$\dot{z} = y - \alpha z - yz$$



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Conclusions and Current (future) Work

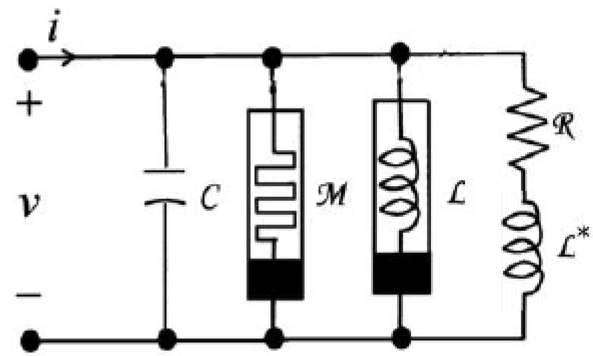
I. Conclusions:

1. We obtained a circuit that uses only *three fundamental circuit elements (only one active)* to obtain chaos.
2. We can pick our choice of nonlinearity, we discussed one particular choice.

II. Current (future) work:

1. Work with colleagues at the University of Western Australia (related to the ideal memristor: cos(phi) term in the Josephson Junction)

Current (future) Work



The normalized circuit equations are Eqs.(1) through (3)

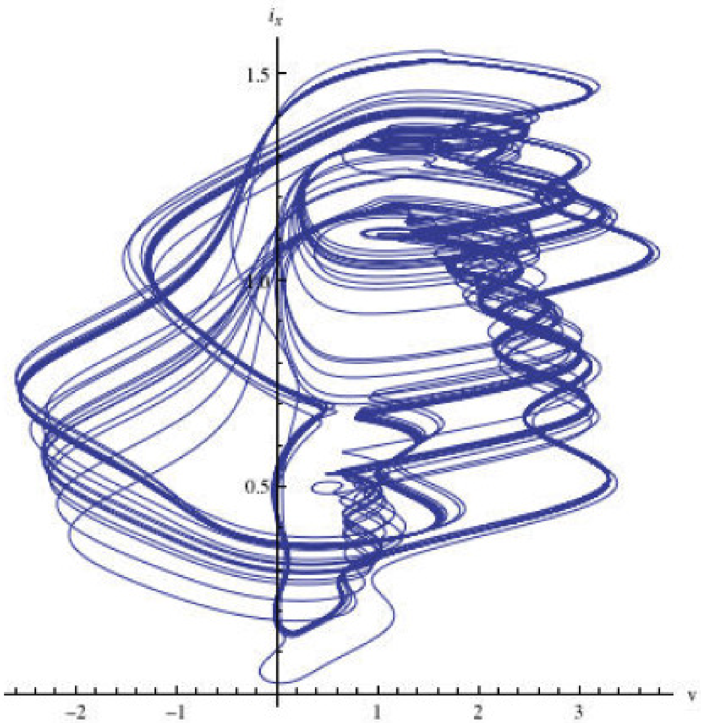
$$\dot{\phi} = v \tag{1}$$

$$\dot{v} = \frac{1}{C}\left(i - G\cos(k_0\phi)v - I_0\sin(\phi) - i_x\right) \tag{2}$$

$$\dot{i_x} = \frac{1}{L^*}\left(v - i_x R\right) \tag{3}$$

The model above was obtained from considering the microscopic theory of Josephson junctions [4]. In Josephson's original papers dealing with thin-film junctions, the coefficients G, k_0 and I_0 in Eq.(2) are dependent on junction voltage [4]. However, this dependence may be neglected provided the voltage stays small when compared to the energy-gap voltage of the individual superconductors [1] Josephson mentions that the cos term in Eq.(2) contributes to damping effects. Although a similar circuit model was proposed in [2], the shunt inductive branch is not included.

Current (future) Work contd.



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Questions?