Chaos In Memristive Model of Josephson Junction

Bharathwaj Muthuswamy, Jovan Jevtic, Fatma Yildrim Dalkiran and Clint Sprott

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In this work, we propose to analyze the model shown in Figure 1 for the Josephson junction. The capacitor C models junction capacitance, the flux-controlled memristor M models the interference among quasi-particle pairs, the nonlinear inductor L is the standard junction current[1]. The shunt composed of the linear resistor R and linear inductor L is used in high frequency applications. This inductive shunt is part of the more accurate RCLSJ model of the Josephson junction [3].

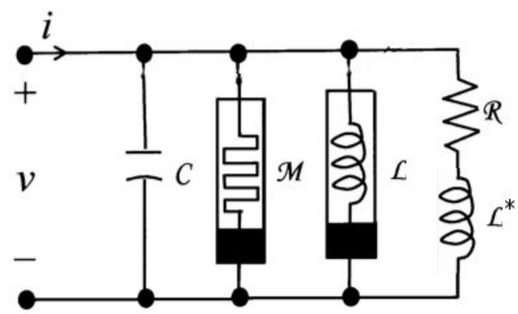

Figure 1: A proposed model for the Josephson junction. Passive sign convention is used for all current-voltage relationships. Current through the memristor is given by $G\cos(k_0\phi)$ and current through the nonlinear inductor is given by $I_0\sin(\phi)$. $\phi = \frac{h}{4\pi e}\gamma$ where h is Planck's constant, e is magnitude of electron charge, γ is the phase difference of the superconducting order parameter across the junction [5].

[1] Note that in most lumped circuit models of the Josephson junction, the nonlinear inductor is usually represented by a pair of triangles [3]. However, the correct symbol that should be used is that of a nonlinear inductor.

The normalized circuit equations are Eqs.(1) through (3)

$$\dot{\phi} = v \tag{1}$$

$$\dot{v} = \frac{1}{C}\left(i - G\cos(k_0\phi)v - I_0\sin(\phi) - i_x\right) \tag{2}$$

$$\dot{i_x} = \frac{1}{L^*}\left(v - i_x R\right) \tag{3}$$

The model above was obtained from considering the microscopic theory of Josephson junctions [4]. In Josephson's original papers dealing with thin-film junctions, the coefficients G, k_0 and I_0 in Eq.(2) are dependent on junction voltage [4]. However, this dependence may be neglected provided the voltage stays small when compared to the energy-gap voltage of the individual superconductors [1] Josephson mentions that the cos term in Eq.(2) contributes to damping effects. Although a similar circuit model was proposed in [2], the shunt inductive branch is not included.

A phase-plot of the simulated attractor obtained from Eqs.(1) through (3) is shown in Fig 2.

References

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[2] Chua, L. O., *Nonlinear Circuit Foundations for Nanodevices, Part I : The Four-Element Torus.* Proceedings of the IEEE (invited paper), Vol. 91, No. 11, pp. 1830 - 1859, 2003.

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[5] Whan, C. B. and Lobb, C. J. *Complex Dynamical Behavior in RCL-shunted Josephson tunnel junctions.* Physical Review E, Vol. 53, No. 1, pp. 405 - 417, 1996.

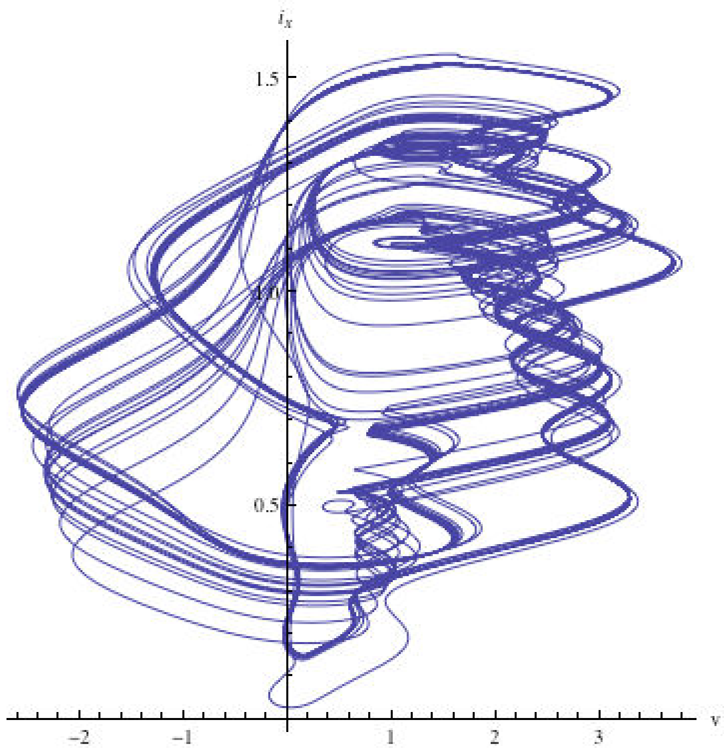

Figure 2: Plot of $(v(t), i_x(t))$. Initial conditions are $\phi(0) = 0, v(0) = 1.25, i_x(0) = 1.4$. Parameters are $C = 1, i = 1, G = 1, k_0 = 2, I_0 = 1, L = 8, R = 1$. Simulation was carried out for 10000 steps using an explicit Euler method in Mathematica 8.