# Simplest [1] (LCM) Chaotic Circuit

Vellore Institute of Technology

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MSOE

[1]Simplest in terms of abstract circuit topology, not number of devices in physical realization

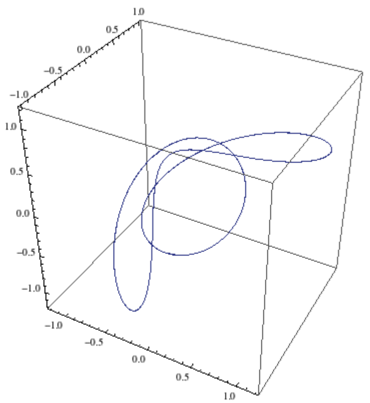
# What do I work on?

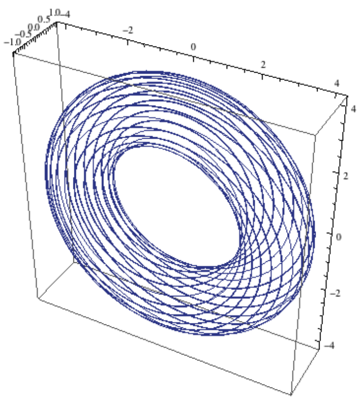
## Nonlinear Dynamical Systems and Embedded Systems

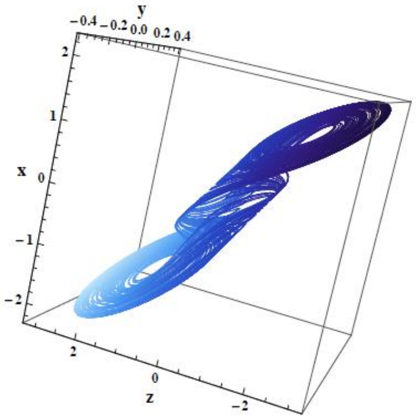
- Applications and Mathematical properties of the LCM chaotic circuit
    - Number Theory (Bharathidasan University, Tiruchy, India)
    - Local activity (University of Western Australia, Perth, Australia)
    - Flow manifolds (I.U.T. de Toulon, La Garde Cedex, France)
- Applications of Chaotic Delay Differential Equations using Field Programmable Gate Arrays (University Putra Malaysia, Malaysia )
- Pattern Recognition Using Cellular Neural Networks (University of California, Berkeley, USA)
- Gait generation using nonlinear dynamics for children with Cerebral Palsy* (Medical College of Wisconsin, Wauwutosa, USA)
- Memristive behavior in superconductors (University of California, Berkeley, USA; Vellore Institute of Technology*, Vellore, India)

## Education

- edX program (University of California, Berkeley; Massachusetts Institute of Technology; Harvard University, USA)
- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺ )

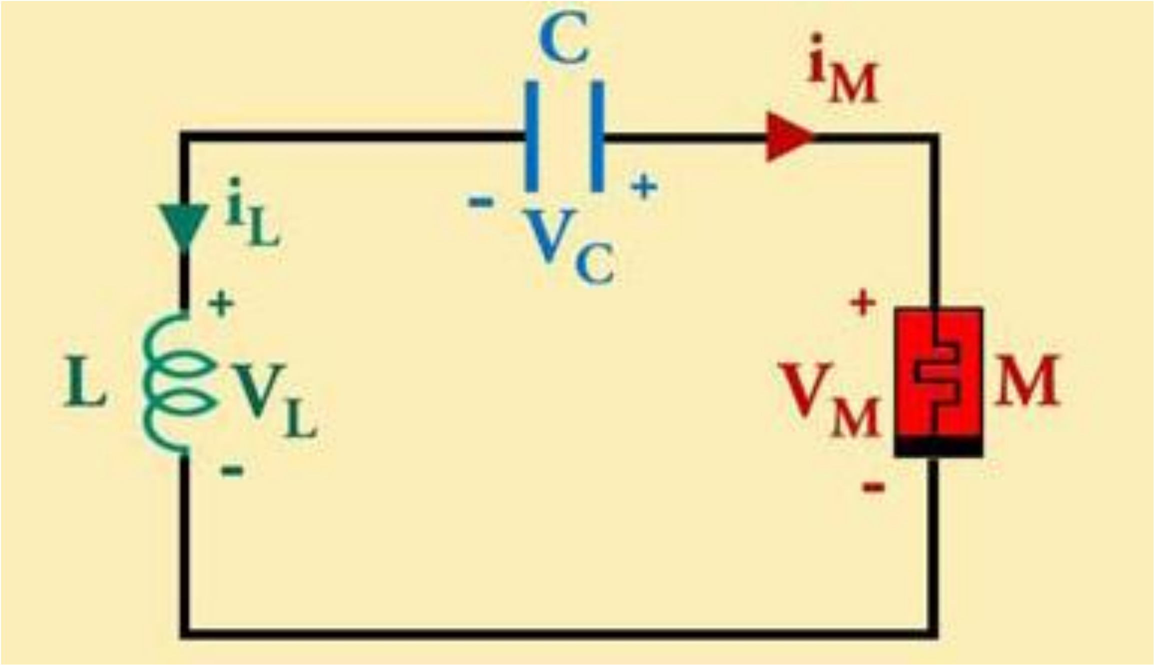






# Goal of This Talk

Obtain chaos in the circuit [5] below:



# Outline

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  1. First course in circuit theory
  2. First course in differential equations

II. Background
  1. Introduction to Chaos
  2. The fundamental circuit elements

III. The Memristor
  1. Properties of the memristor
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# Introduction to chaos

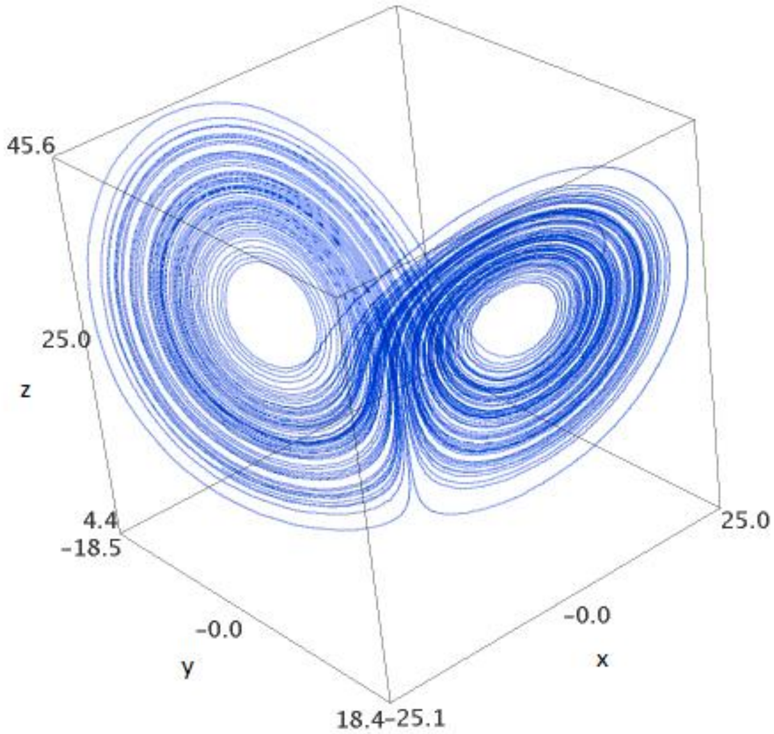
- "Birth" of Chaos: Lorenz Attractor [6]
  - Edward Lorenz introduced the following nonlinear system of differential equations as a crude model of weather in 1963:

$$\dot{x} = -\sigma \cdot x + \sigma \cdot y$$
$$\dot{y} = \rho \cdot x - y - x \cdot z \qquad (1)$$
$$\dot{z} = -\beta \cdot z + x \cdot y$$

Parameters: $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

ICs: $x_0 = 10, y_0 = 20, z_0 = 30,$

Simulation time: 100 seconds




  - Lorenz discovered that model dynamics were extremely sensitive to initial conditions and the trajectories were aperiodic but bounded.
  - But, does chaos exist *physically?* Answer is: YES. For example, chaotic circuits by Sprott [7].

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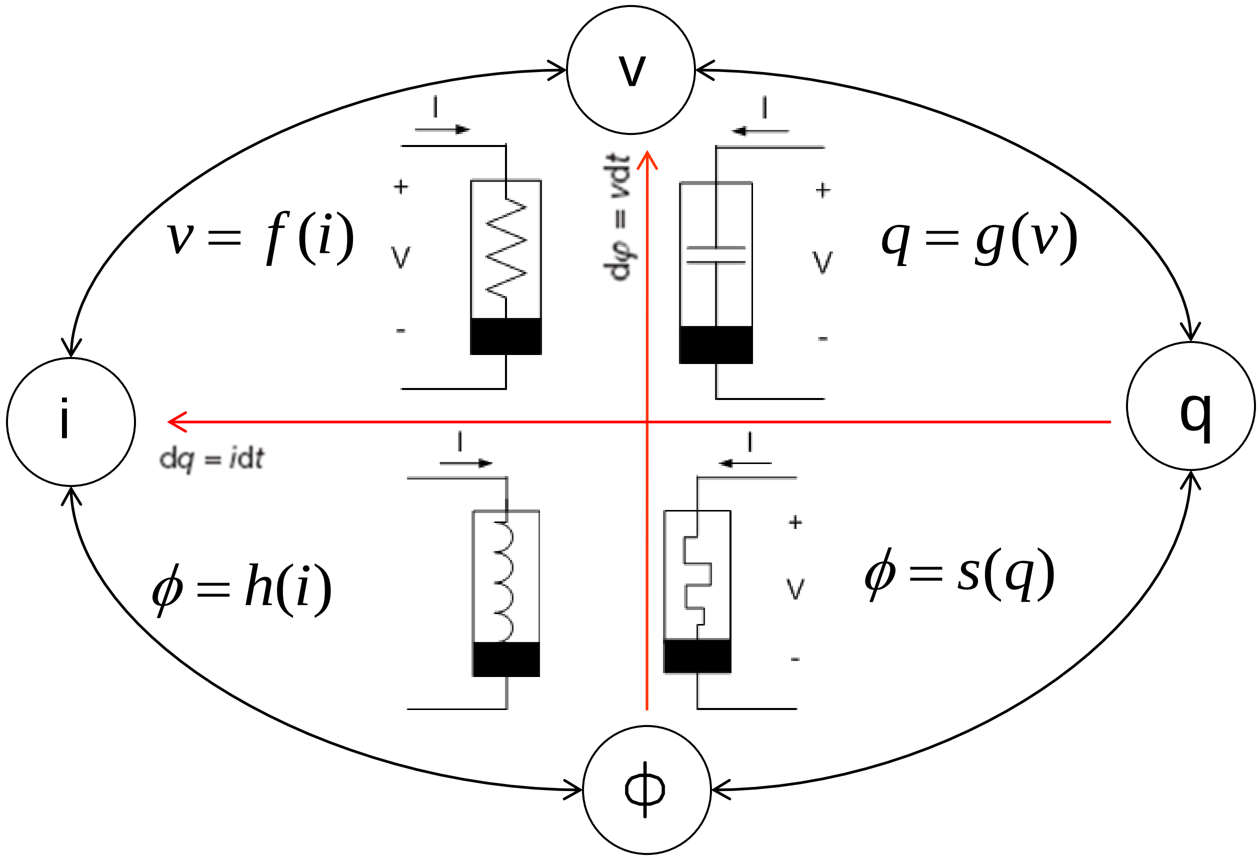
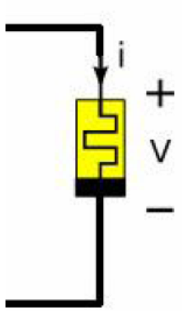
# The Fundamental Circuit Elements




Memristors were first postulated by Leon. O Chua in 1971 [2]

# Properties of the Memristor [2]

Circuit symbol:



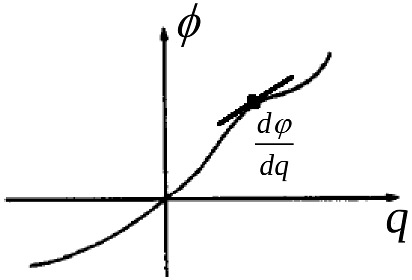

A memristor defines a *relation* of the from: $g(\phi, q) = 0 \quad (2)$

If *g* is a single-valued function of charge (flux), then the memristor is charge-controlled (flux-controlled)

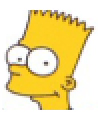
Memristor i-v relationship:

*M(q(t))* is the incremental memristance

$$v(t) \triangleq \frac{d\phi}{dt} = \frac{d\phi}{dq}\frac{dq}{dt} \triangleq M(q(t))i(t) \quad (3)$$

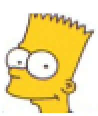



Q1: Why is the memristor called “memory resistor”?



Because of the definition of memristance: $v(t) = M(q(t))i(t) = M\left(\int_{-\infty}^{t} i(\tau)\right)i(t)$

Q2: Why is the memristor not relevant in linear circuit theory?



1. If M(q(t)) is a constant: $v(t) = M(q(t))i(t) = Mi(t) = Ri(t)$
2. Principle of superposition is not* applicable:

$$M\left(\int_{-\infty}^{t}(i_1+i_2)(\tau)\right)(i_1+i_2)(t) = M\left(\int_{-\infty}^{t}(i_1)(\tau) + \int_{-\infty}^{t}(i_1)(\tau)\right)(i_1+i_2)(t) \neq M\left(\int_{-\infty}^{t}(i_1)(\tau)\right)i_1(t) + M\left(\int_{-\infty}^{t}(i_2)(\tau)\right)i_2(t)$$

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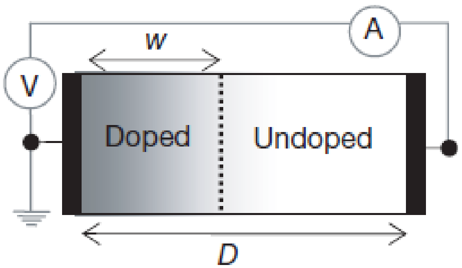
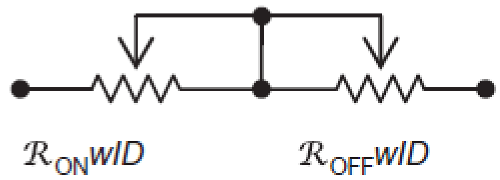
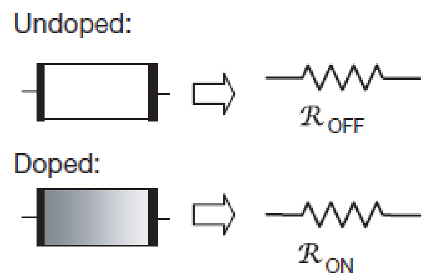
# Hewlett-Packard's memristor [9]

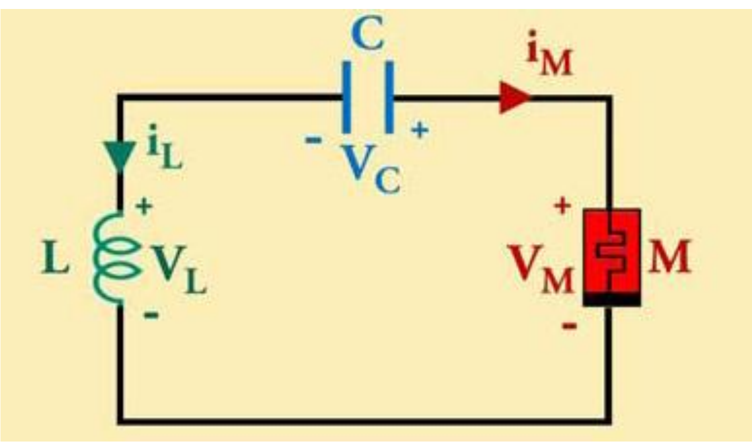



$$v(t) = \left( \mathcal{R}_{ON} \frac{w(t)}{D} + \mathcal{R}_{OFF} \left( 1 - \frac{w(t)}{D} \right) \right) i(t) \xrightarrow[\mathcal{R}_{ON} \ll \mathcal{R}_{OFF}]{} M(q) = \mathcal{R}_{OFF} \left( 1 - \frac{\mu_V \mathcal{R}_{ON}}{D^2} q(t) \right)$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \mu_V \frac{\mathcal{R}_{ON}}{D} i(t)$$




Circuit equations:

$$\dot{v}_c = \frac{i_L}{C}$$

$$\dot{q}_M = -i_L$$

$$\xrightarrow{x \triangleq v_c,\, z \triangleq q_M,\, y \triangleq i_L}$$

System equations:

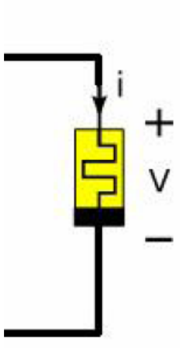
$$\dot{x} = \frac{y}{C}$$

$$\dot{z} = -y \qquad (5)$$

$$\dot{y} = \frac{-1}{L}\left(x + M(z)y\right)$$

$$v_L + v_C = v_M \Rightarrow L\frac{di_L}{dt} = -v_C + M(q_M)i_M \Rightarrow i'_L = \frac{-1}{L}\left(v_C + M(q_M)i_L\right)$$

# Memristive Devices [3]



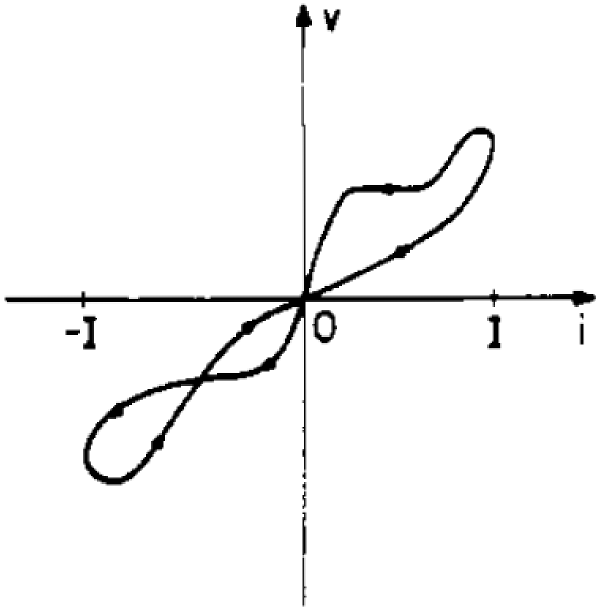

$$v \triangleq R(z,i)i$$
$$\dot{z} = f(z,i) \qquad (6)$$

$$\xrightarrow{z \triangleq q,\, R(z,i) \triangleq M(q)}$$

$$v \triangleq M(q)i$$
$$\dot{q} = i$$

The functions *R* and *f* are defined as:

$$R : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}$$
$$f : \mathbb{R}^1 \times \mathbb{R} \to \mathbb{R}^1$$



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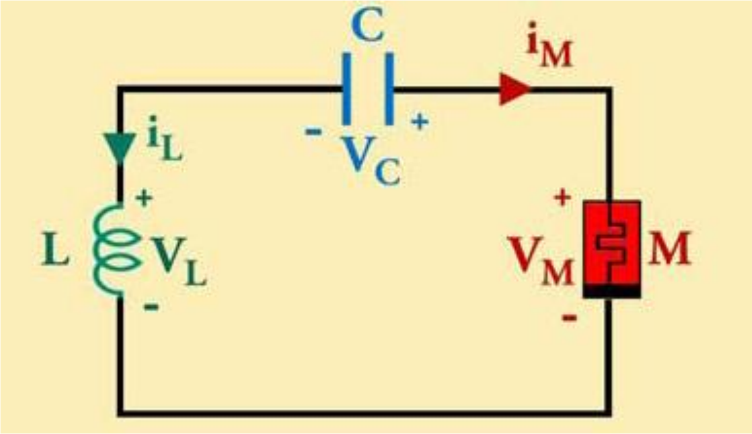
# Derivation of Circuit Equations [5]

$$v_M \triangleq R(z, i_M) i_M$$
$$\dot{z} = f(z, i_M)$$




Circuit equations:

$$\dot{v}_c = \frac{i_L}{C}$$
$$i'_L = \frac{-1}{L}\left(v_C + R(z, i_L) i_L\right)$$
$$\dot{z} \triangleq f(z, i_L)$$

$$\xrightarrow{x \triangleq v_c,\, y \triangleq i_L}$$

System equations:

$$\dot{x} = \frac{y}{C}$$
$$\dot{y} = \frac{-1}{L}\left(x + R(z, y) y\right) \quad (7)$$
$$\dot{z} = f(z, y)$$

Specifically:

$$\dot{x} = \frac{y}{C}$$
$$\dot{y} = \frac{-1}{L}\left(x + \beta(z^2 - 1) y\right) \quad (8)$$
$$\dot{z} = -y - \alpha z + yz$$

Parameters:

$$\xrightarrow{\phantom{xxxxxx}}$$
$$C = 1, L = 3$$
$$\beta = \frac{3}{2}, \alpha = \frac{3}{5}$$

$$\dot{x} = y$$
$$\dot{y} = \frac{-x}{3} - \frac{z^2 y}{2} + \frac{y}{2} \quad (9)$$
$$\dot{z} = -y - 0.6z + yz$$

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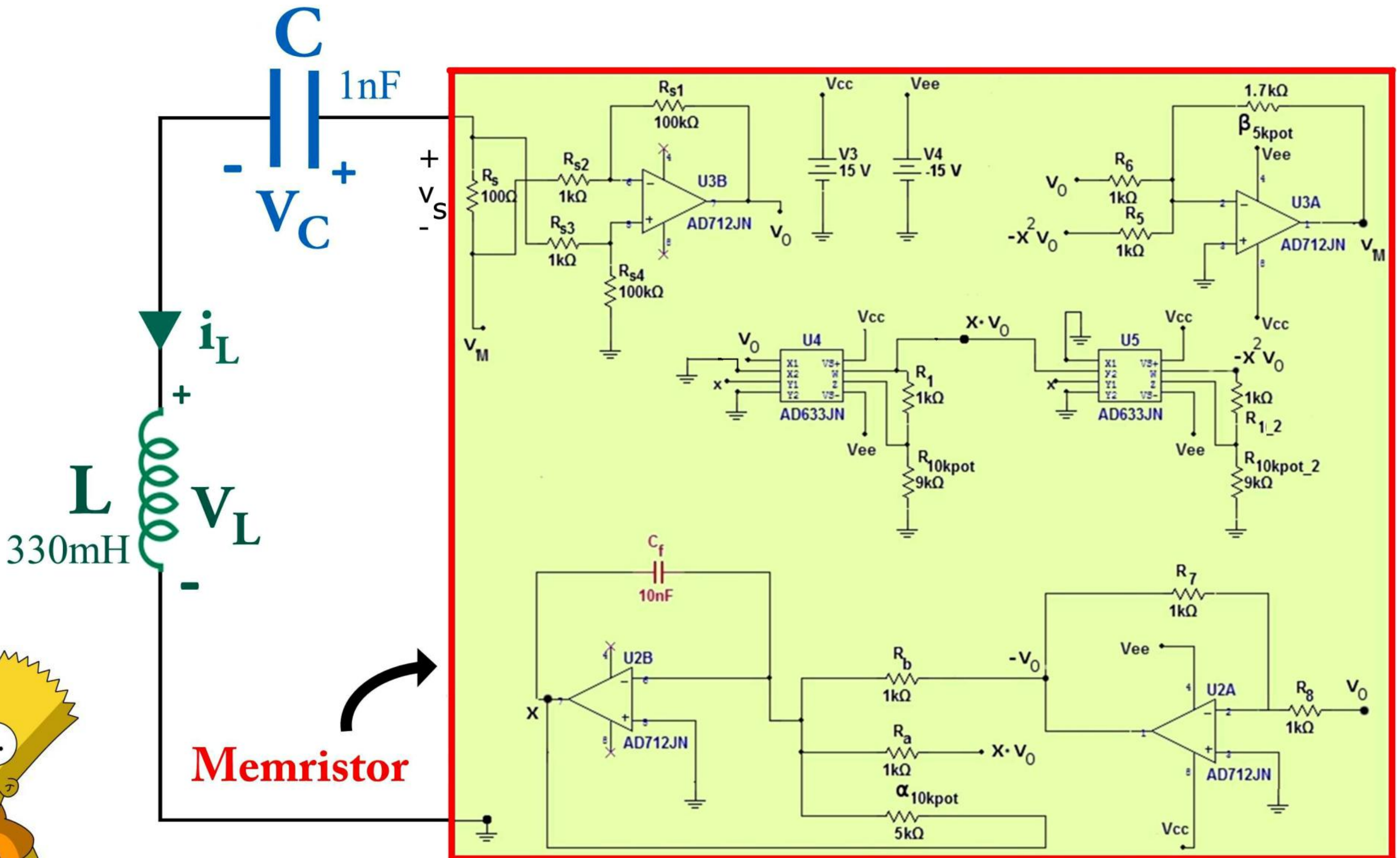
# Physical Realization of the Memristor [5]

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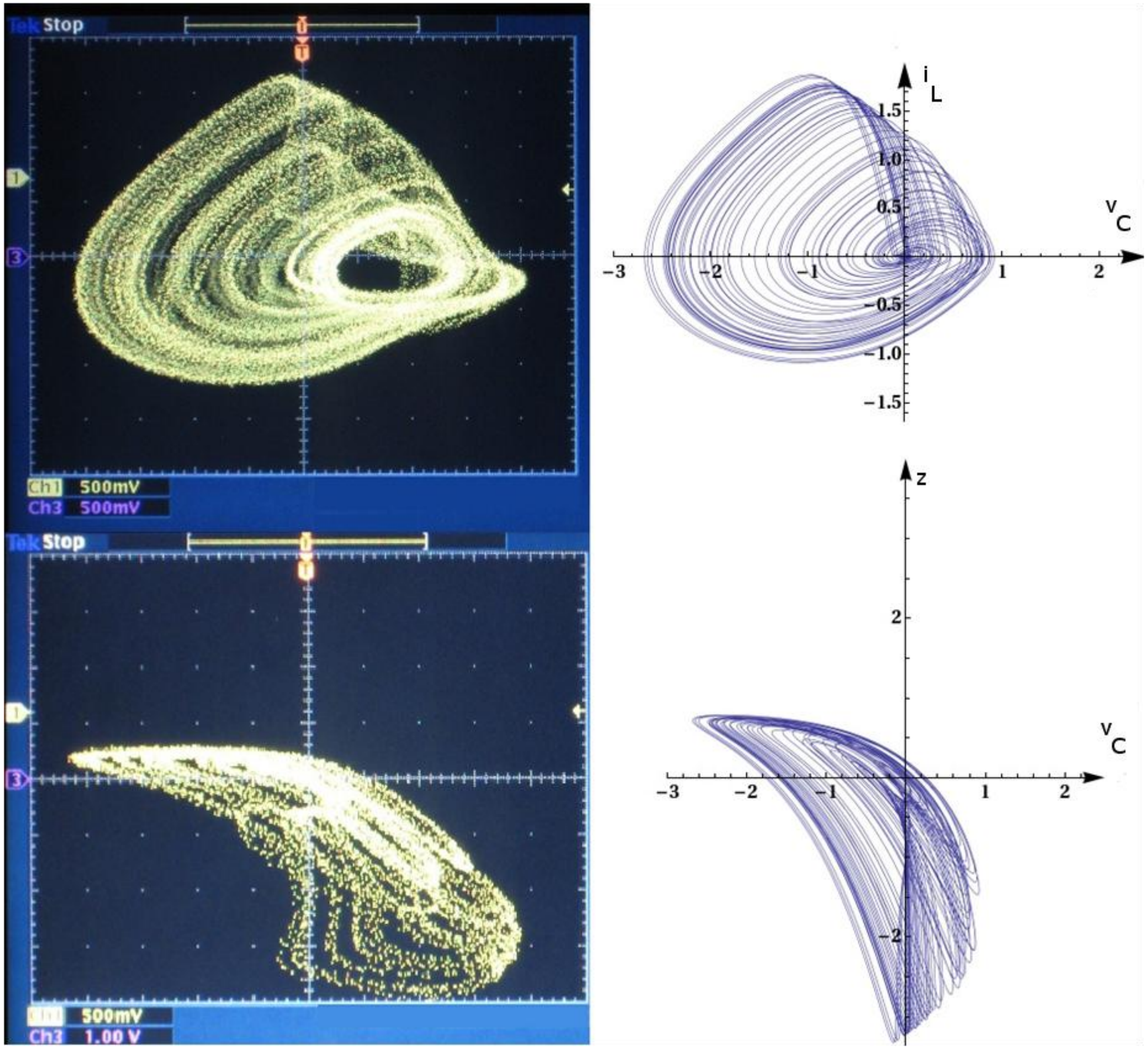
# Attractors from the Circuit [5]

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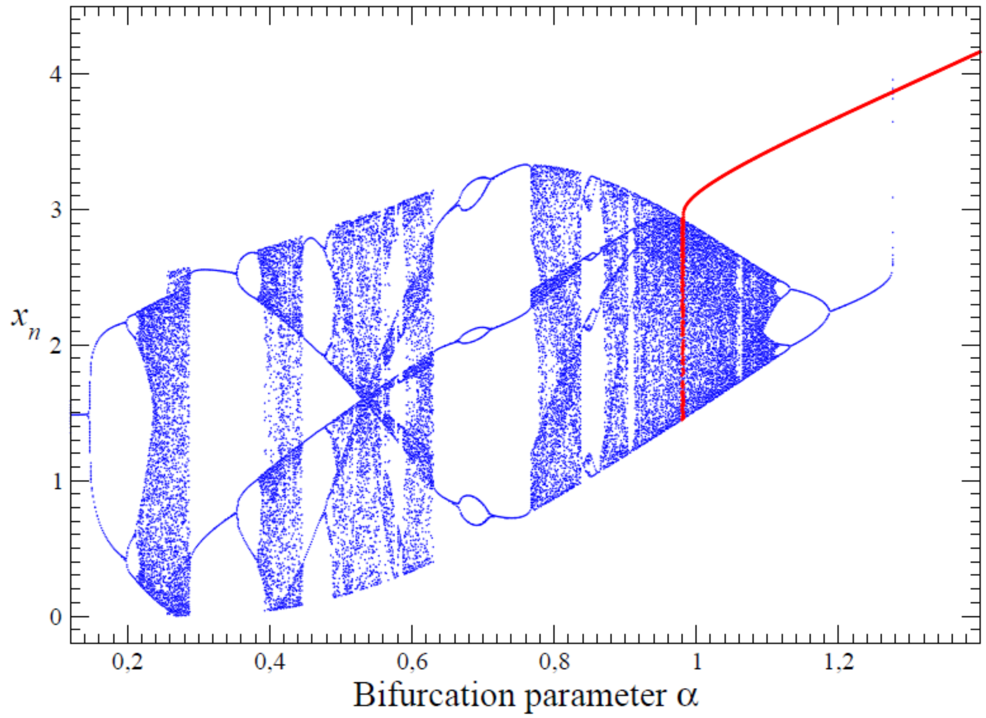
# Rigorous Mathematical Analysis

Paper by Ginoux et. al. "Topological Analysis of Chaotic Solution of Three-Element Memristive Circuit". International Journal of Bifurcation and Chaos, Vol. 20, No. 11., pp. 3819 – 3829, Nov. 2010.

$$\dot{x} = y$$

$$\dot{y} = -\frac{x}{3} + \frac{y}{2} - \frac{yz^2}{2}$$

$$\dot{z} = y - \alpha z - yz$$



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# Conclusions and Future Work

I. Conclusions:

1. We obtained a circuit that uses only *three fundamental circuit elements (only one active)* to obtain chaos.
2. We can pick our choice of nonlinearity, we discussed one particular choice.

II. Future work:

1. Physical memristor from the Josephson Junction

# References


1. Alligood, K. T., Sauer, T. and Yorke, J. A. *Chaos: An Introduction to Dynamical Systems*. Springer, 1997.
2. Chua, L. O. "Memristor-The Missing Circuit Element". *IEEE Transactions on Circuit Theory*, Vol. CT-18, No. 5, pp. 507- 519. September 1971.
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6. Lorenz, E. N. "Deterministic Nonperiodic Flow". *Journal of Atmospheric Sciences*, vol. 20, pp/ 130-141, 1963.
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8. Strogatz, S. H. *Nonlinear Dynamics and Chaos*. Addison-Wesley, 1994.
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# Questions?