Nonlinear Dynamical Embedded Systems
The XUP Approach

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Xilinx, San Jose, California

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PhD Advisor: Dr. Leon O. Chua (co-advised by Dr. Pravin P. Varaiya)

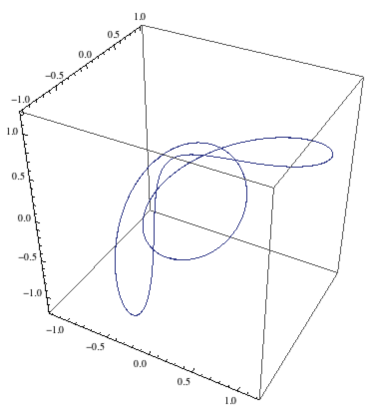
What do I work on?

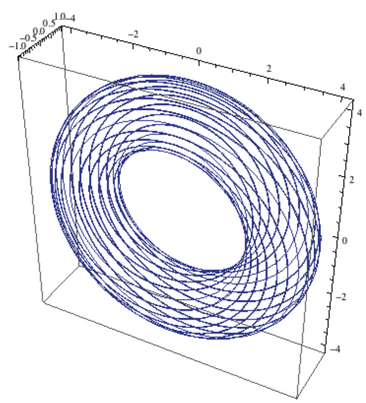
Nonlinear Dynamical Systems and Embedded Systems

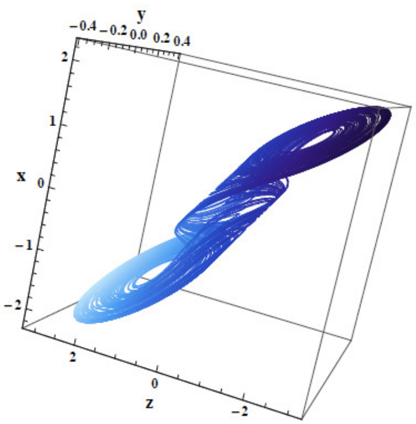
- Nonlinear Dynamics on FPGAs
    - Chaotic Differential Equations (VIT; University Putra Malaysia, Malaysia)
    - Pattern Recognition Using Cellular Neural Network (VIT; University of Oklahoma School of Medicine)
- Physical Memristors: discharge tubes, PN junctions and Josephson Junctions
  (IIT Chennai; Vellore Institute of Technology (VIT), Vellore, India)
- Applications and Mathematical properties of the Muthuswamy-Chua system
  (VIT; AGH-University of Science and Technology, Poland)
- Quantum Chaos*
  (VIT)

Education

- Nonlinear Dynamics at the undergraduate level (with folks from all over the world ☺)







Goals of this Talk

1. Roadmap for Making Zynq (FPGAs) an Ubiquitous Teaching/Research Platform
2. How we plan to use the Zynq in AY 2015

But, before we get started…

“FDGA”

“Field Debugging of the Gate Array”

☺

Outline

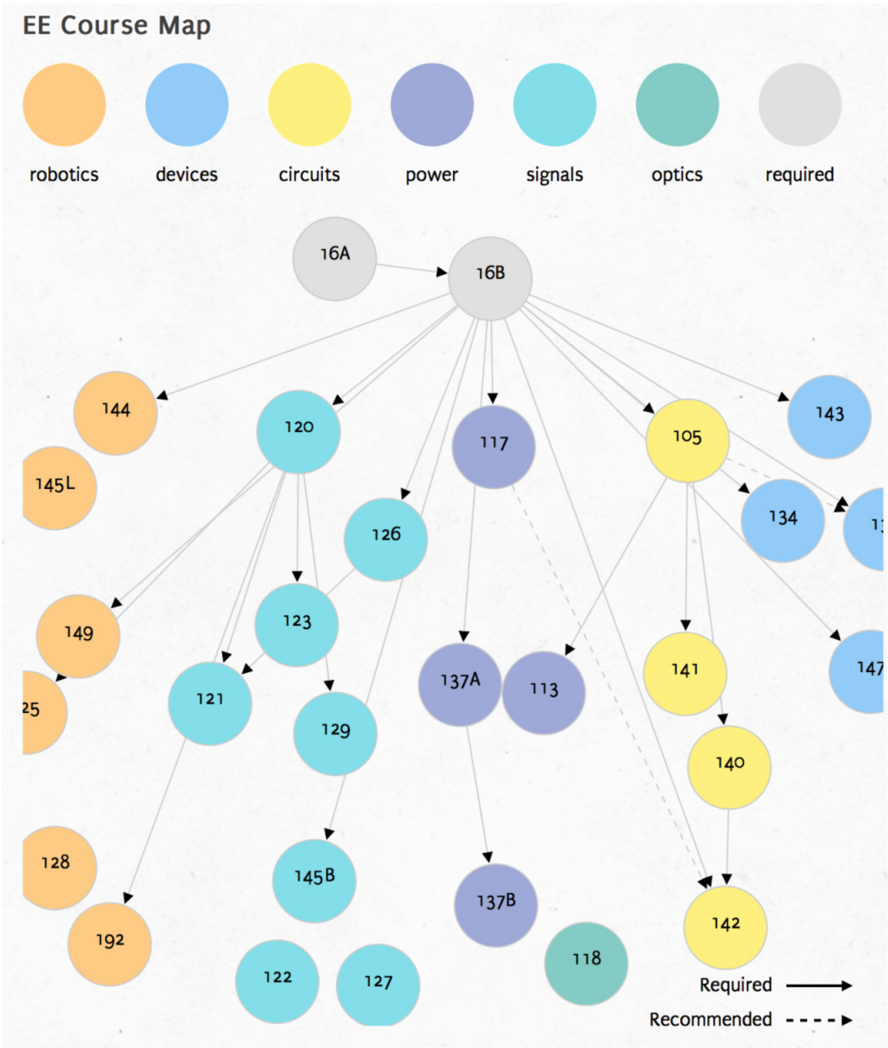
I. The Road Map Illustrated

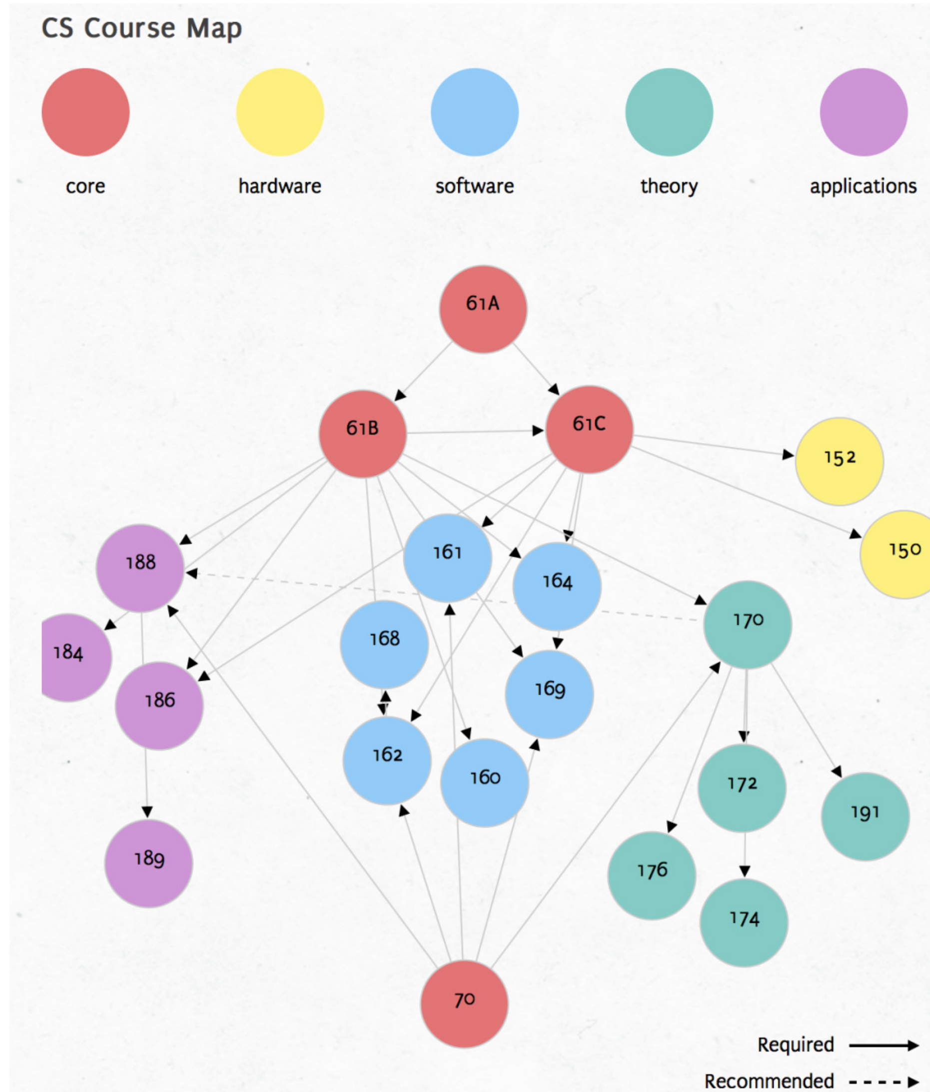
II. FPGA Based Nonlinear (Chaotic) Dynamics on the ZedBoard
  1. Overall Block Diagram
  2. Block Diagram for Implementing Chaotic Systems
  3. Examples: Lorenz, Chen, Ikeda
  4. LIVE DEMO: The Muthuswamy-Chua System

III. Nonlinear Dynamical Embedded Systems
  1. Proposal: Nonlinear control on FPGA
  2. FPGAs in the sciences
    a. Current Work: Nonlinear Dynamics of Infant ROP and Atrial Fibrillation
    b. Current Work: Pattern Recognition using Cellular Nonlinear Networks
    c. Future Work: Emulation of PDEs
  3. Proposal: NetFPGA (?)

IV. Conclusions and References

GO BEARS : The Roadmap Illustrated





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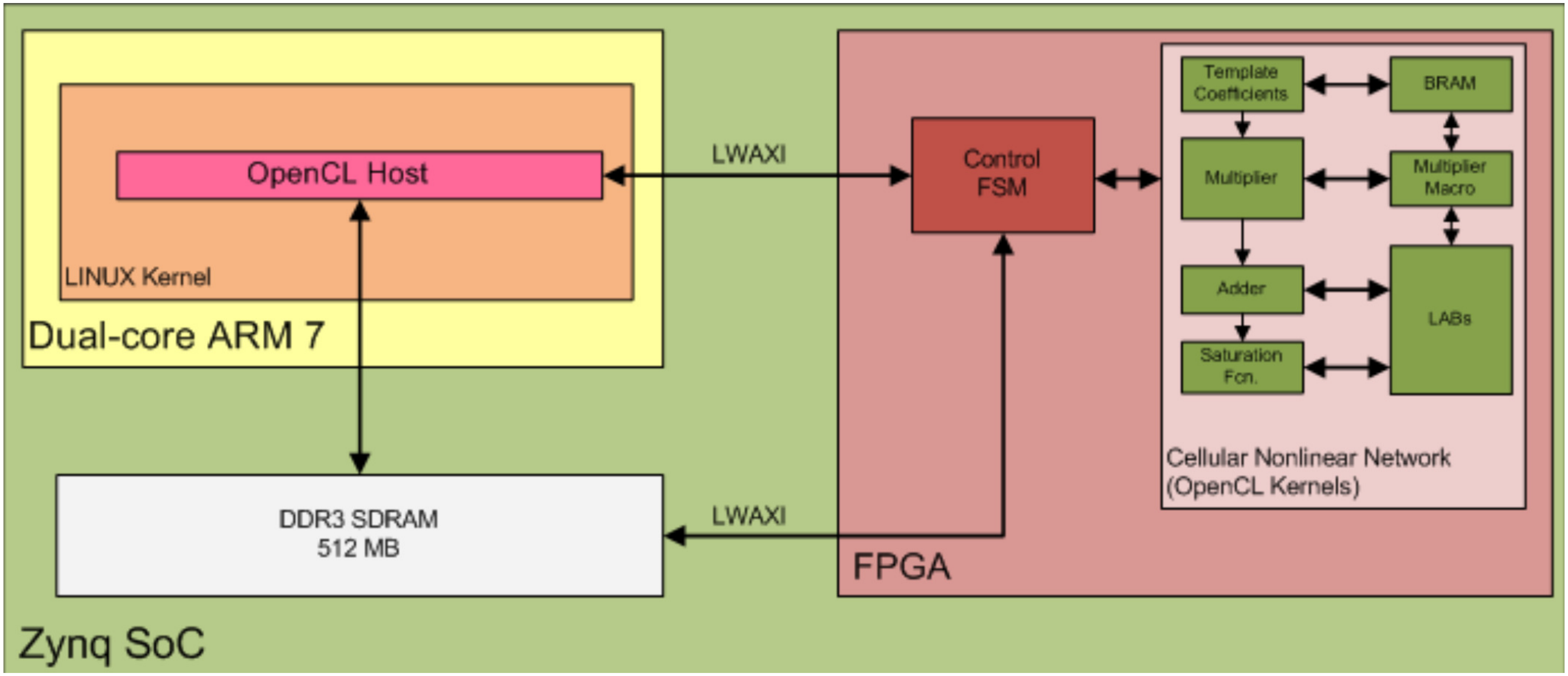
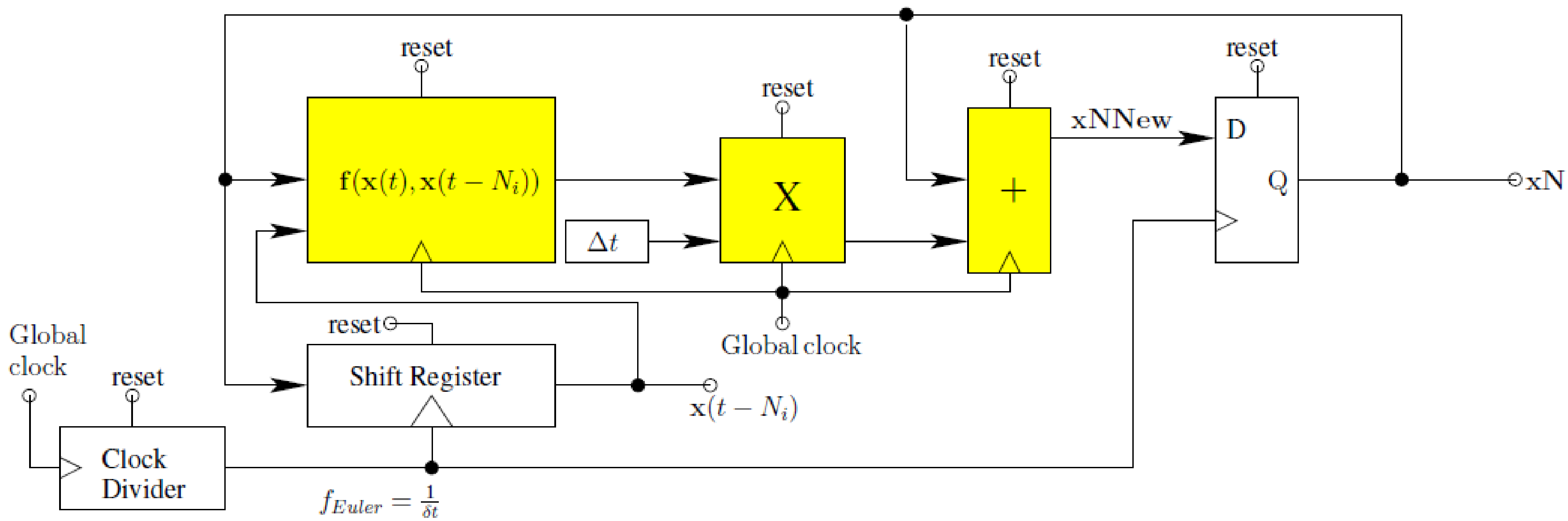
Overall Block Diagram



Block Diagram for Implementing Chaotic Systems [10]



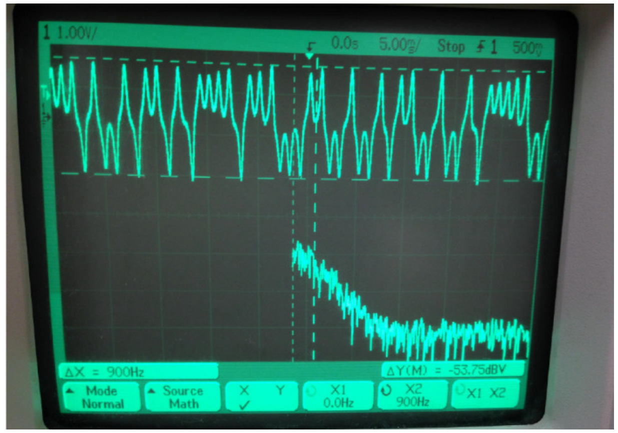
Example 1 : Lorenz [10]

$$\begin{aligned} \dot{x} &= -\sigma x + \sigma y \\ \dot{y} &= -xz + \rho x - y \qquad (1) \\ \dot{z} &= xy - \beta z \end{aligned}$$

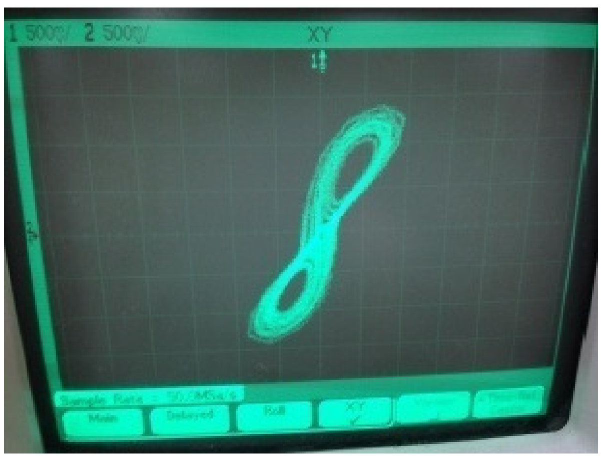
Parameters:

$$\sigma = 10, \rho = 28, \beta = \frac{8}{3}$$

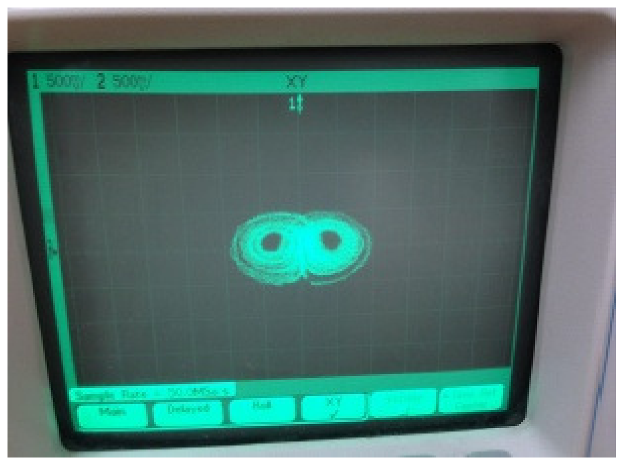
i.cs: $(10, 20, 30)$



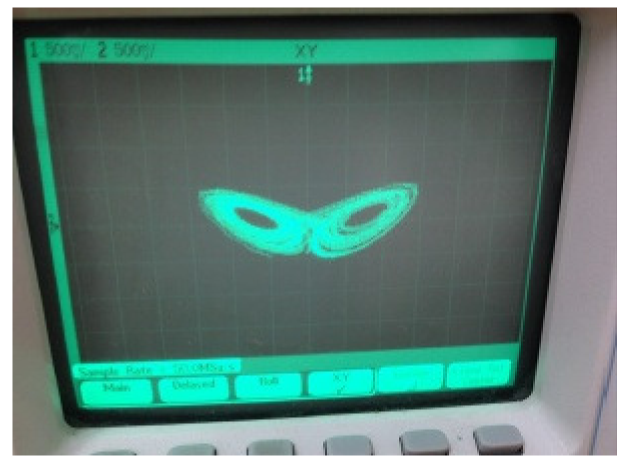
Time Domain Waveforms and FFT



y(t) vs x(t)



z(t) vs y(t)



z(t) vs x(t)

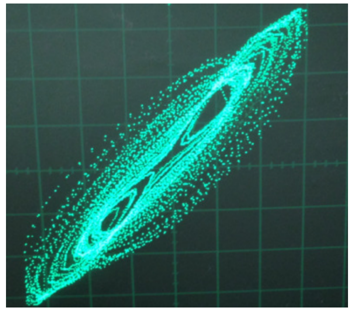
Example 2 : Chen (ModelSim) [10]

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - xz + cy \qquad (2) \\ \dot{z} &= xy - bz \end{aligned}$$

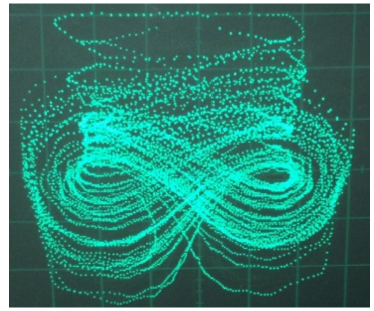
Parameters:

$a = 35, b = 3, c = 28$

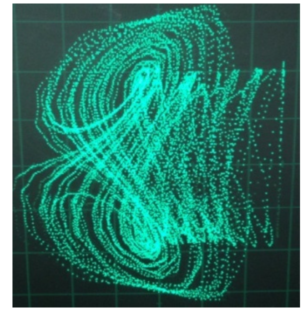
i.cs: $(10, 20, 30)$



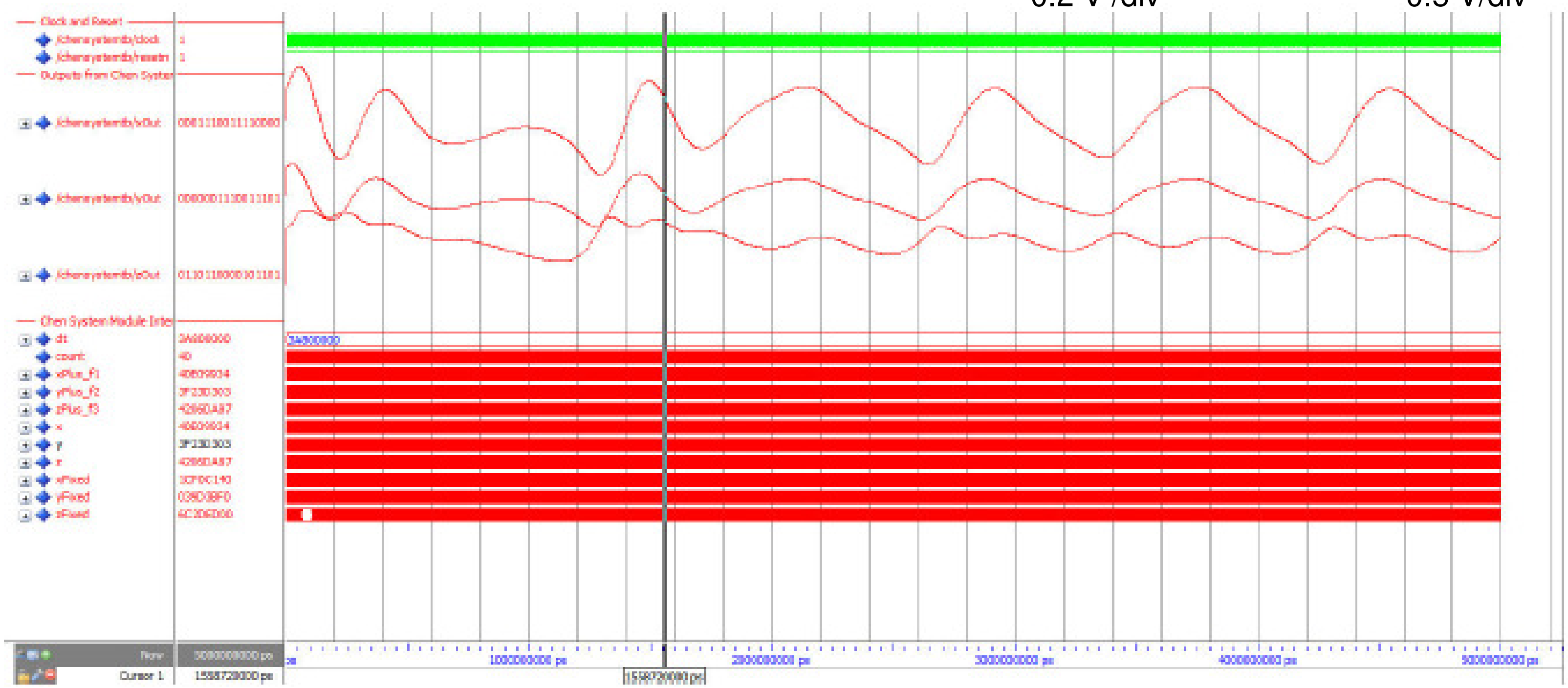
y(t) vs x(t), 0.5 V/div



z(t) vs y(t), 0.5 V/div; 0.2 V /div



x(t) vs z(t), 0.2 V/div; 0.5 V/div



Functional simulation in ModelSim for the Chen chaotic system, with analog interpretation of audio codec input

The Ikeda Delay Differential Equation [10]

$$\dot{x} = \mu \sin(x(t-\tau)) - \alpha x(t) \qquad (3)$$

Parameters: $\mu=6, \tau=1, \alpha=0.1$

i.cs: $x(t \leq 0) = 0.1$

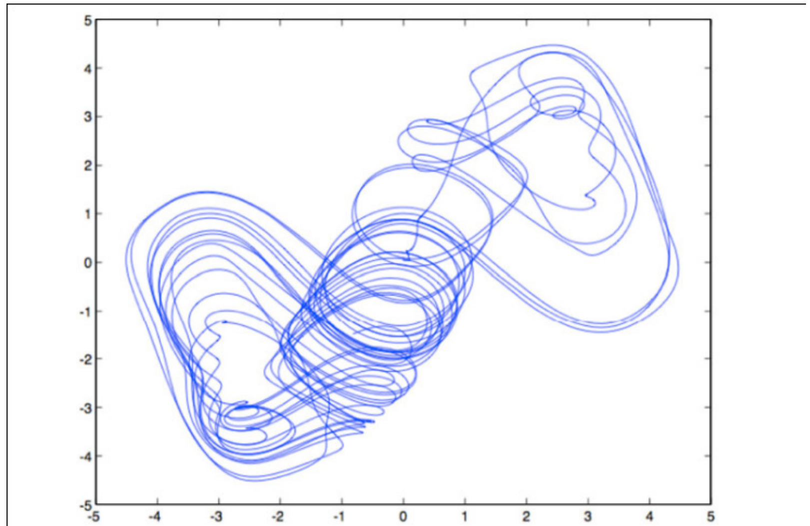
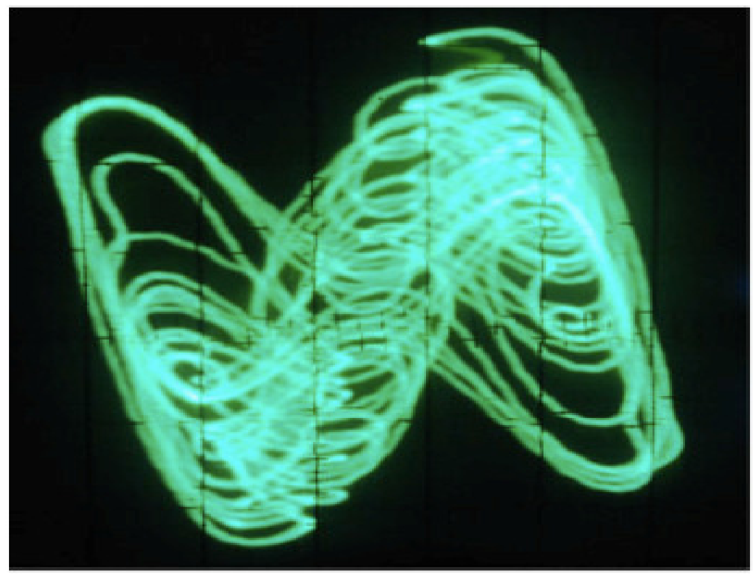


Fig. 5. Result from hardware co-simulation, plotted using XY graph in Simulink. y-axis is $x(t)$, $x - axis$ is $x(t-\tau)$.



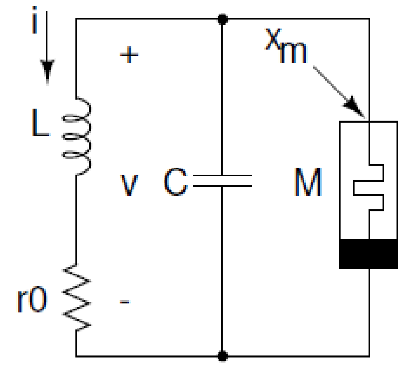
The Muthuswamy-Chua Oscillator [under review]
LIVE DEMO!

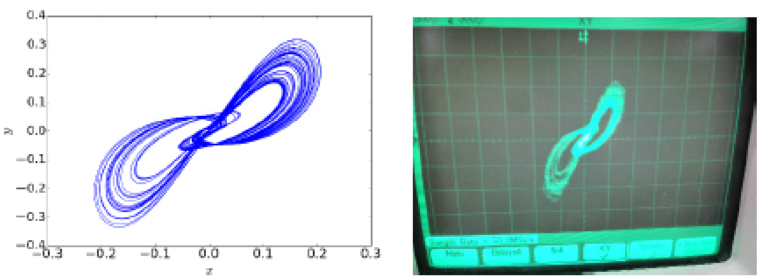
$$\dot{x} = \frac{y - x \cdot r_0}{L}$$
$$\dot{y} = \frac{-1}{C}(x + \alpha z y) \qquad (4)$$
$$\dot{z} = -\beta + y^2$$

Parameters:

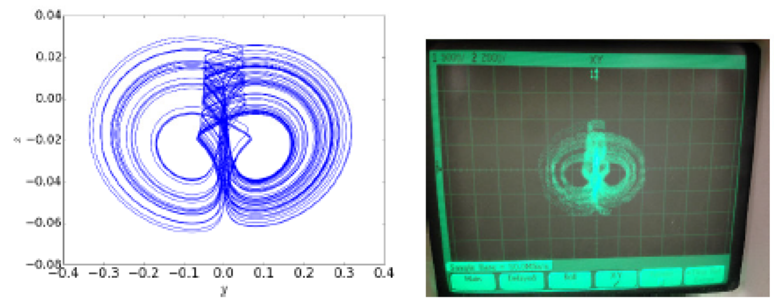
$L = 1/40, r_0 = 1, C = 1/10, \alpha = 33/40, \beta = 10$

i.cs: (0.1, 0, 0.1)

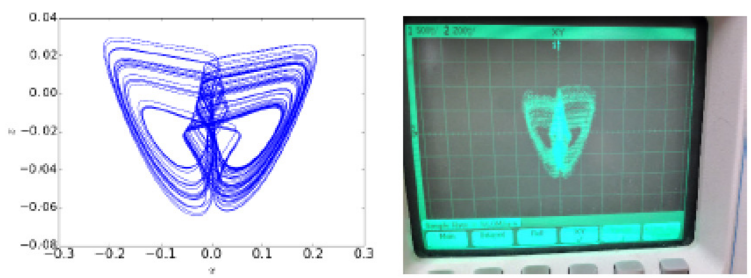




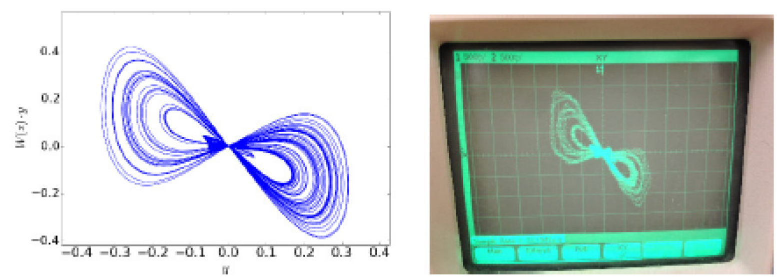
(a) y vs x. Scope axes scales are 0.5 V/div.



(b) z vs y. Scope x-axis scale is 0.5 V/div, y-axis scale is 0.2 V/div.



(c) z vs x. Scope x-axis scale is 0.5 V/div, y-axis scale is 0.2 V/div.



(d) Pinched-hysteresis loops, the fingerprint [4] of a memristor. Scope axes scale are 0.5 V/div.

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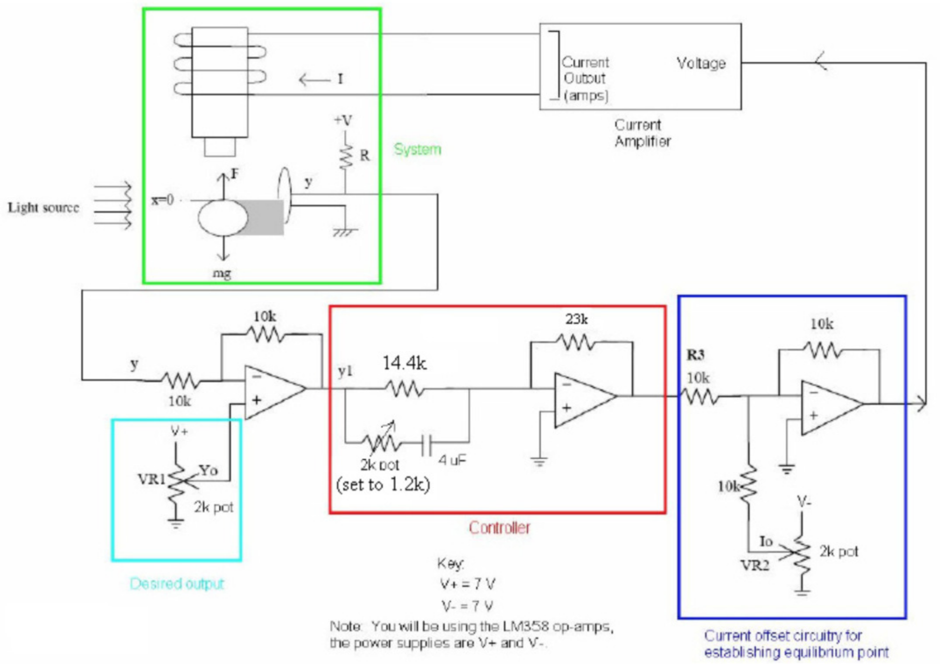
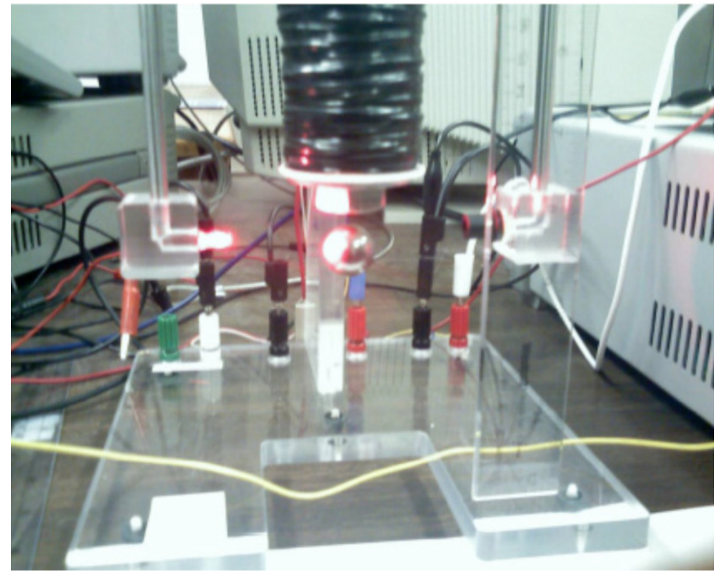
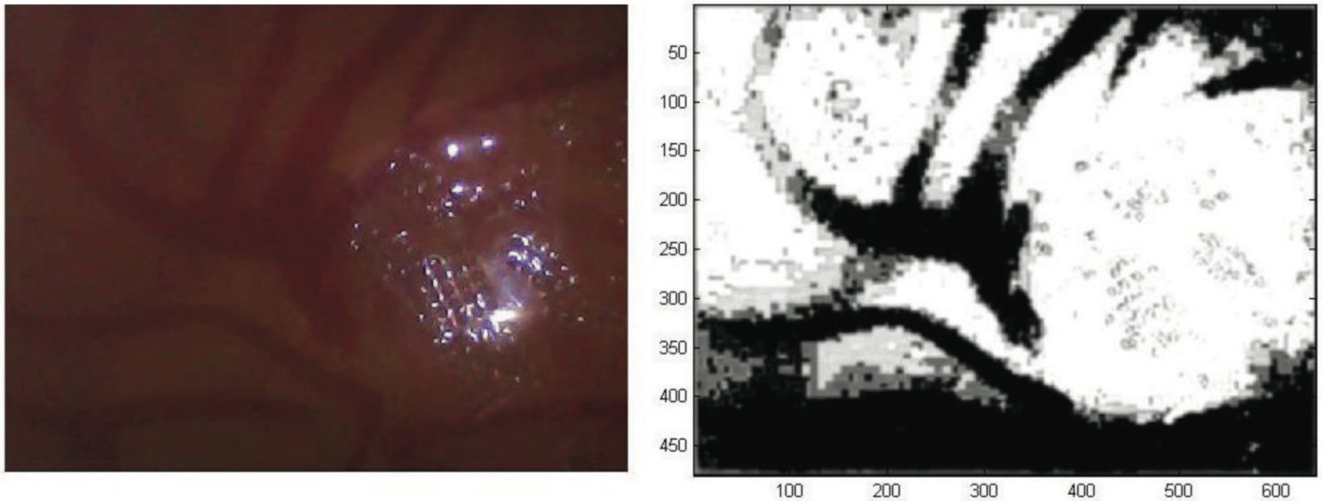
Proposal: Nonlinear Control on FPGA

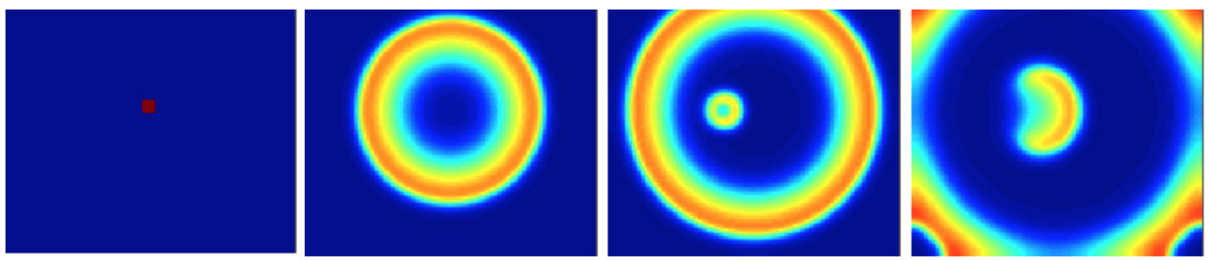




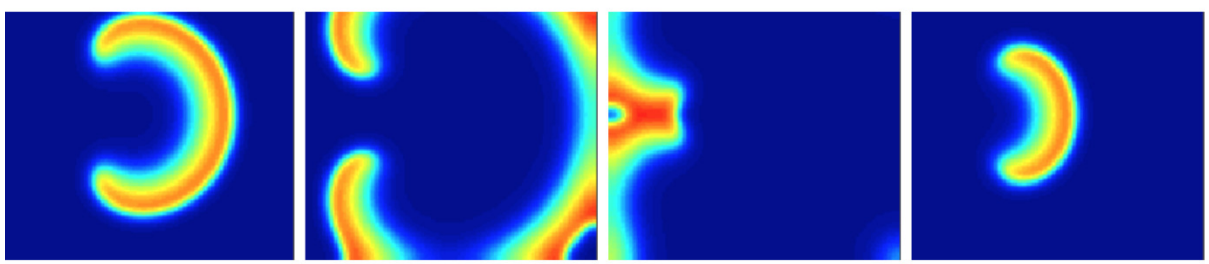
FPGAs in the Sciences:
Nonlinear Dynamics of Infant ROP and Atrial Fibrillation



Input Output

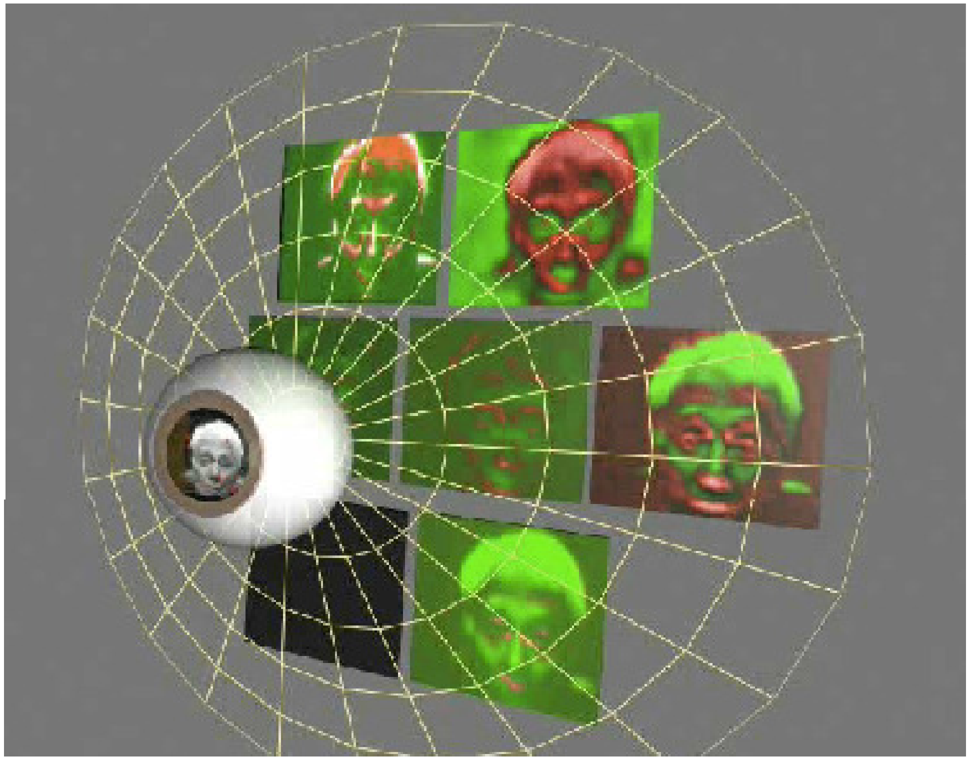


(a) $t = 0$ (b) $t = 7$ (c) $t = 9$ (d) $t = 13$



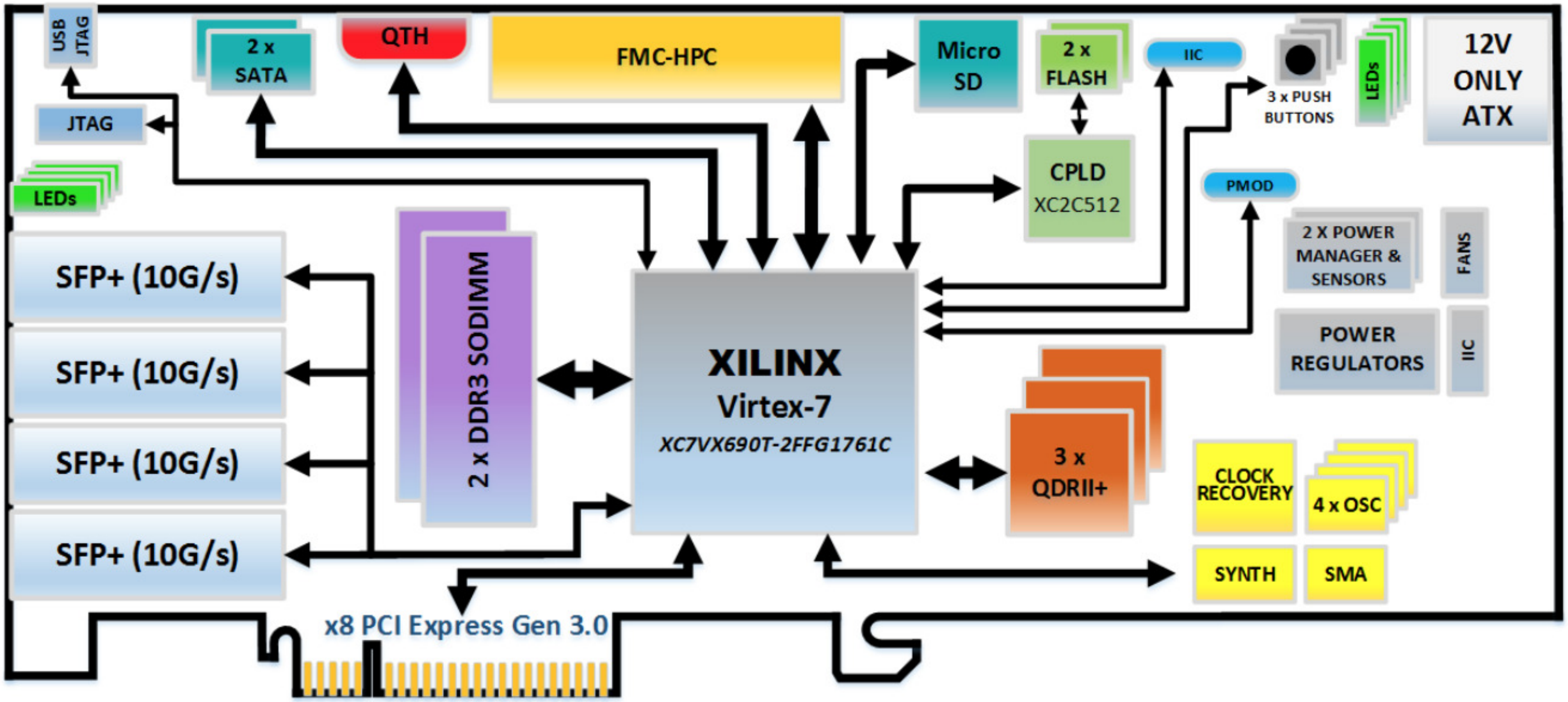
(e) $t = 16$ (f) $t = 21$ (g) $t = 23$ (h) $t = 30$

FPGAs in the Sciences:
Pattern Recognition using Cellular Nonlinear Networks



Utilize ideas behind the retinal hypercircuit (Werblin Lab, Berkeley) for recognition of hand-drawn circuit diagrams

Proposal: NetFPGA (?)



NetFPGA SUME block diagram

Reference: Zilberman, Noa et. al. NetFPGA SUME: Toward Research Commodity 100Gb/s

Availabe, online: http://www.cl.cam.ac.uk/~nz247/publications/zilberman2014sume.pdf

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Conclusions

Overview of my ideas for potential contribution towards XUP:

Nonlinear Dynamical Embedded Systems

1. Current work : FPGA based chaotic dynamics (Vol. 1) and Rigorous methods (Vol. 2). To be published by Springer-Verlag in AY 2015-2016.
2. Introduce FPGAs to students early (freshman) using SDAccel and OpenCL (EECS 16A and 16B)
3. Systems courses can use FPGAs (EECS 120, EECS 122, EECS 127 etc.)
4. Further support CS150/152
5. Robotics and "Embedded" Systems courses (EECS 125, EECS 145L)
6. Continue to grow as a researcher by (ref.: Senior XUP Applications Engineer Job description):
   - Lead technical projects and supervise intern projects as required
   - Create teaching materials, labs, demos and application notes. Identify and propagate examples of best teaching practices/research
   - Deliver technical training workshops
   - Support conferences, trade shows and customer-facing events
   - Support XUP customer queries and technical issues in USA and around the world
   - Work in close partnership with other local Xilinx teams and external partners to support XUP customers
   - Maintain effective relationships with XUP teams around the world
   - Contribute to the specification, evaluation and development of FPGA development systems for use in academia
   - Be familiar with latest developments in the related markets and local industries

References

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MANY THANKS TO DR. JEVTIC AND DR. THOMAS (MSOE)

Questions and Discussion…