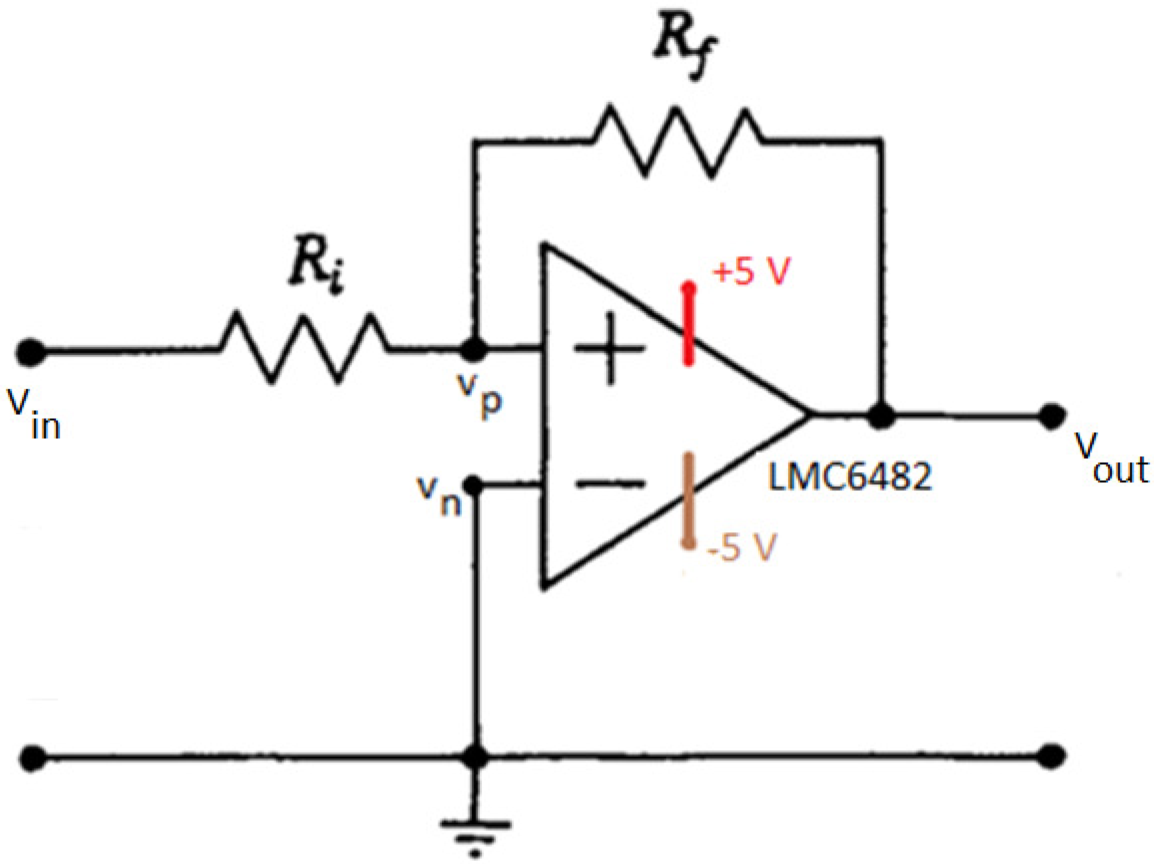
# Fundamental (Nonlinear) Circuit Elements: Positive Feedback And Relaxation Oscillations

**Bharathwaj Muthuswamy**
**Assistant Professor of Electrical Engineering**
**Milwaukee School of Engineering**
**BS (2002); MS (2005); PhD (2009) in EECS from the University of California, Berkeley**
**Research Interests : Nonlinear Dynamics, Embedded Systems and Education**
**muthuswamy@msoe.edu**

**Date : April 15th 2014**

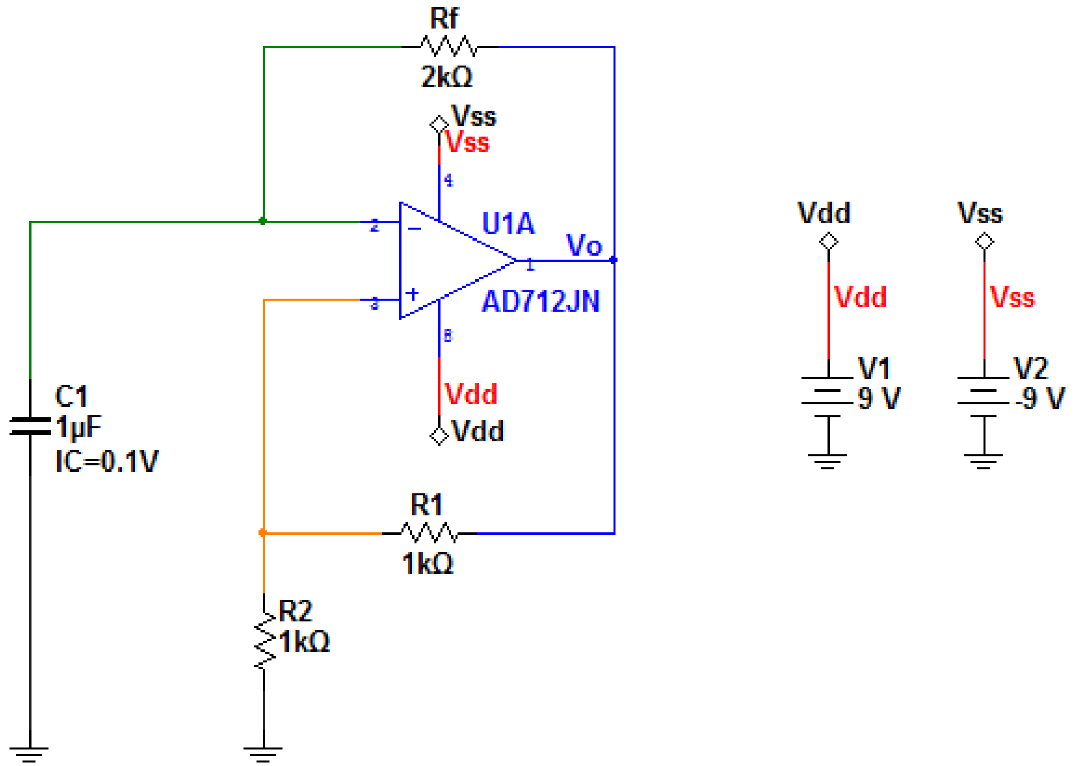
# Goal(s) of Lecture

Sketch $V_{out}$ vs $V_{in}$ for the circuit below. Verify your analysis experimentally. Explain any discrepancies between your analysis and experimental results.

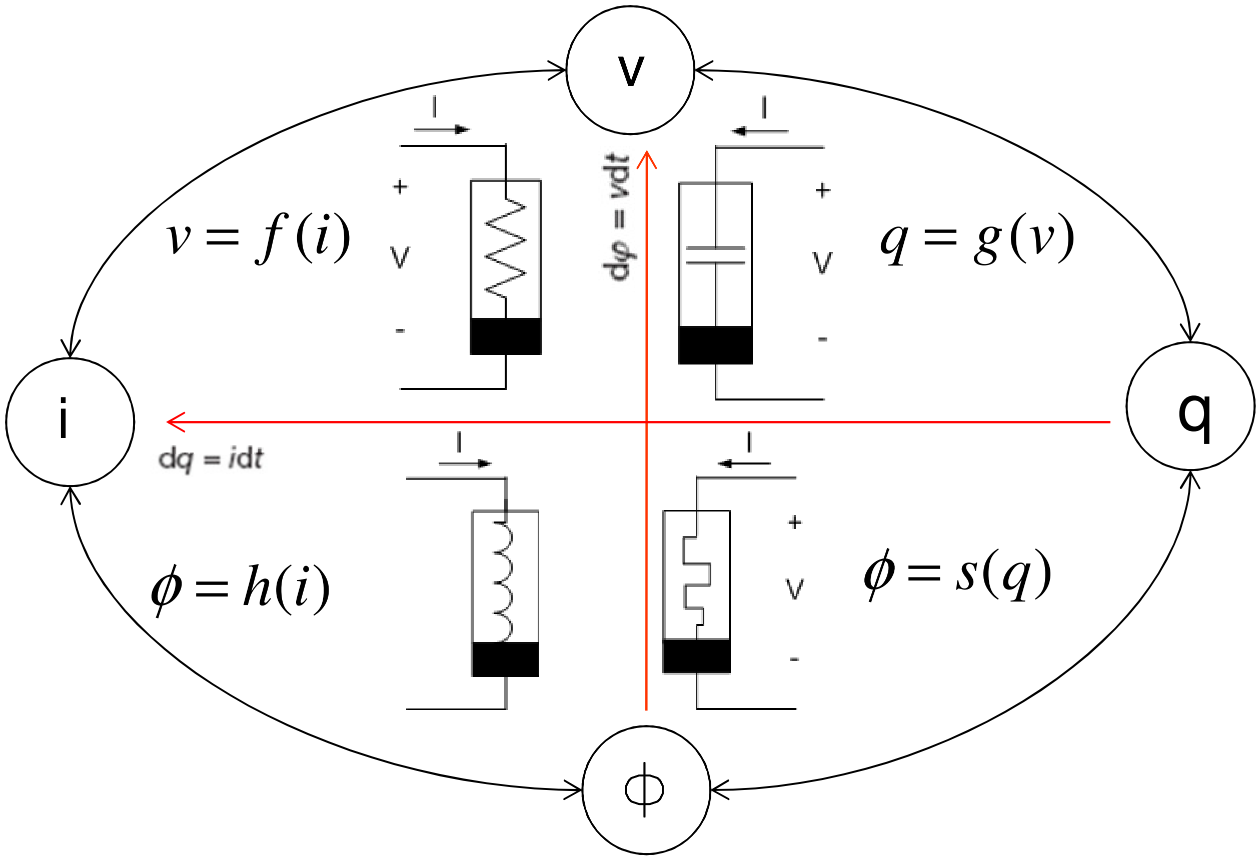


# Goal(s) of Lecture

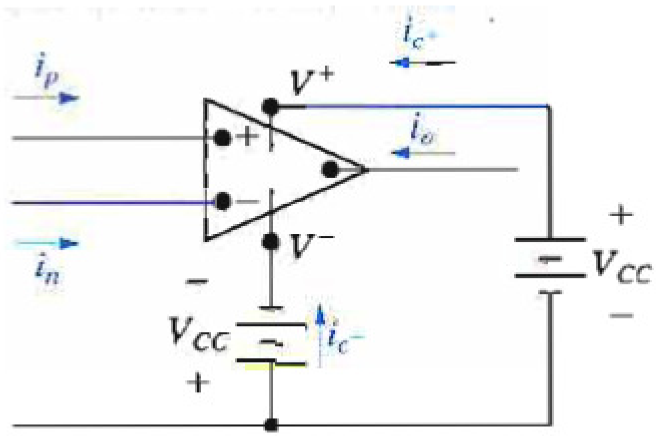
Analytically determine the period and duty cycle in the circuit below.

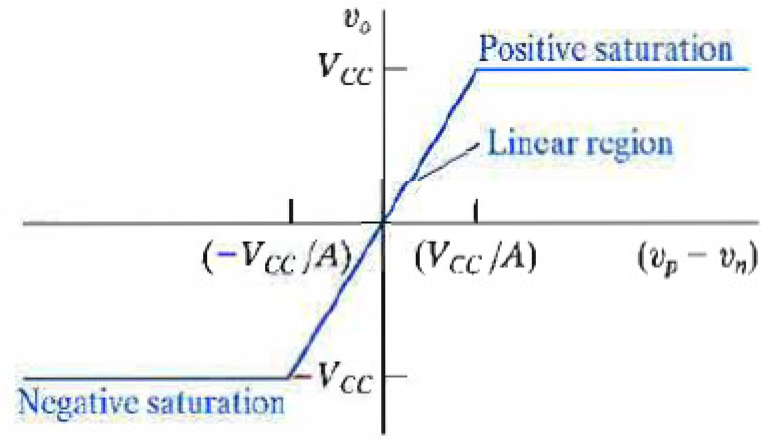


# “Review” : The Fundamental Circuit Elements



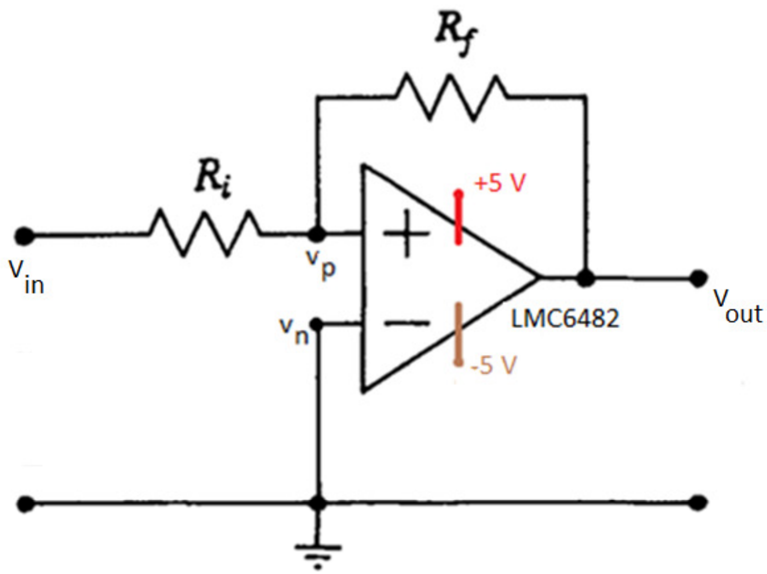
# "Review" : The Operational Amplifier




$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC}, \\ A(v_p - v_n) & -V_{CC} \leq A(v_p - v_n) \leq +V_{CC}, \\ +V_{CC} & A(v_p - v_n) > +V_{CC}. \end{cases}$$

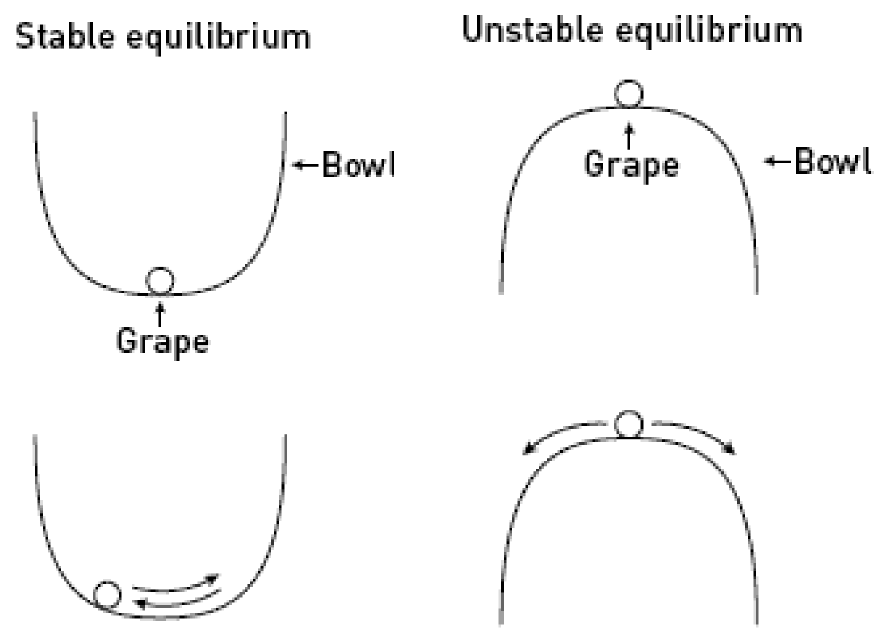
# Step 1: Understand Problem




**This circuit (system) is non-dynamical!**

Equilibrium Points are:

Reference : http://www.learner.org/courses/mathilluminated/units/13/textbook/03.php



# Step 2: Devise a Plan

1. Derive $V_{out}$ vs $V_{in}$ using op-amp golden rules

2. Interpret the results

3. Devise an experiment to confirm our derivation (theory)

# Step 3 (a): Carry out the plan – Derive $V_{out}$ vs $V_{in}$ (linear region)

Assuming op-amp is in the linear region: $v_p = v_n$

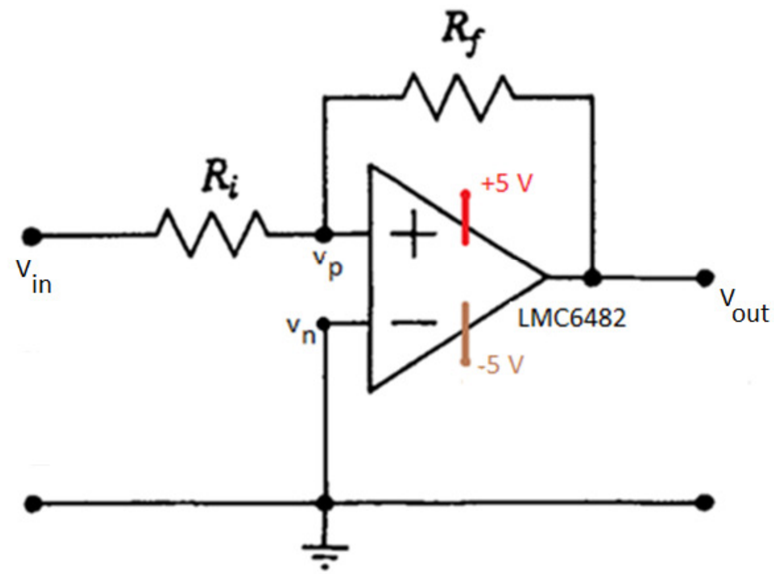
Applying KCL and Ohm's law at $v_p$ :

$$\frac{v_{in} - v_p}{R_i} = \frac{v_p - v_{out}}{R_f}$$

$$\Rightarrow \quad \frac{v_{in} - v_n}{R_i} = \frac{v_p - v_{out}}{R_f}$$

$$\Rightarrow \quad \frac{v_{in}}{R_i} = \frac{-v_{out}}{R_f}$$

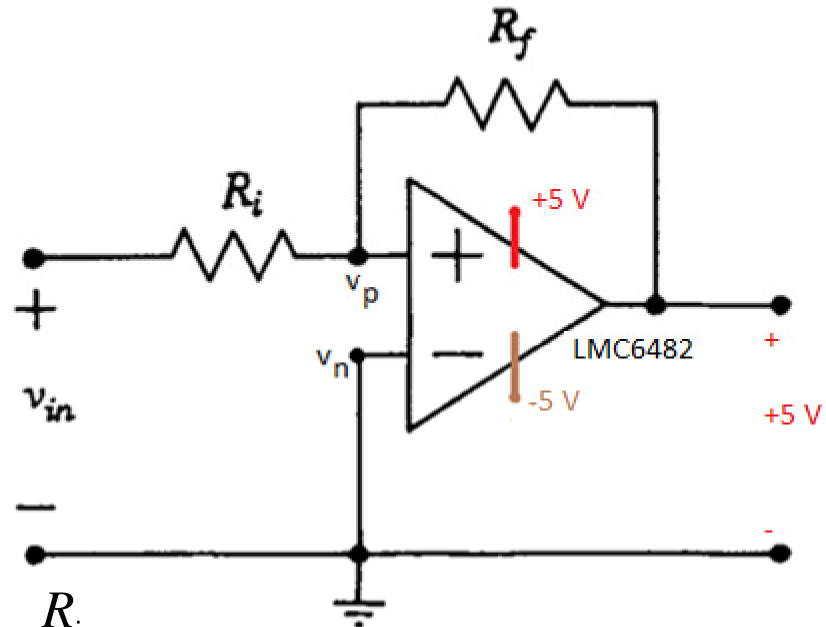
Hence $v_{out} = -2v_{in}$



# Step 3 (b): Carry out the plan – Derive $V_{out}$ vs $V_{in}$ (positive saturation region)

Since op-amp is assumed to be in positive saturation: $v_{out} = +v_{CC}, \; v_p \geq v_n$

Applying KCL and Ohm's law at $v_p$ :




$$\frac{v_{in} - v_p}{R_i} = \frac{v_p - v_{out}}{R_f}$$

$$\Rightarrow \quad v_p = \frac{v_{in} + \frac{R_i}{R_f} v_{CC}}{1 + \frac{R_i}{R_f}}$$

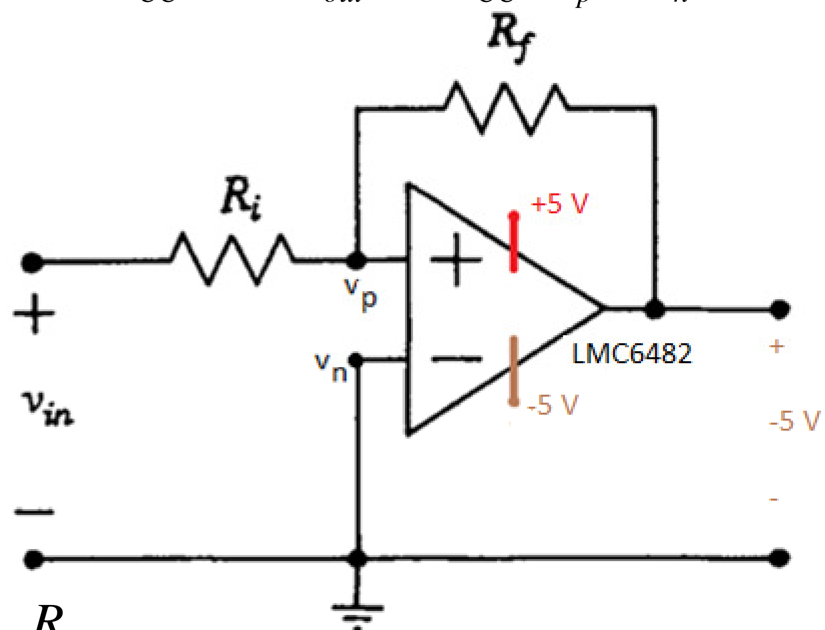
Now, $v_{out} = +v_{CC}$ if $v_p \geq v_n \Rightarrow v_{out} = +v_{CC}$ if $\dfrac{v_{in} + \frac{R_i}{R_f} v_{CC}}{1 + \frac{R_i}{R_f}} \geq 0$

Hence: $v_{out} = 5 \text{ if } v_{in} \geq -2.5 \text{ V}$

# Step 3 (c): Carry out the plan – Derive $V_{out}$ vs $V_{in}$ (negative saturation region)

Since op-amp is assumed to be in negative saturation ($v_{CC}$>0) : $v_{out} = -v_{CC}$, $v_p \leq v_n$

Applying KCL and Ohm's law at $v_p$ :




$$\frac{v_{in} - v_p}{R_i} = \frac{v_p - v_{out}}{R_f}$$

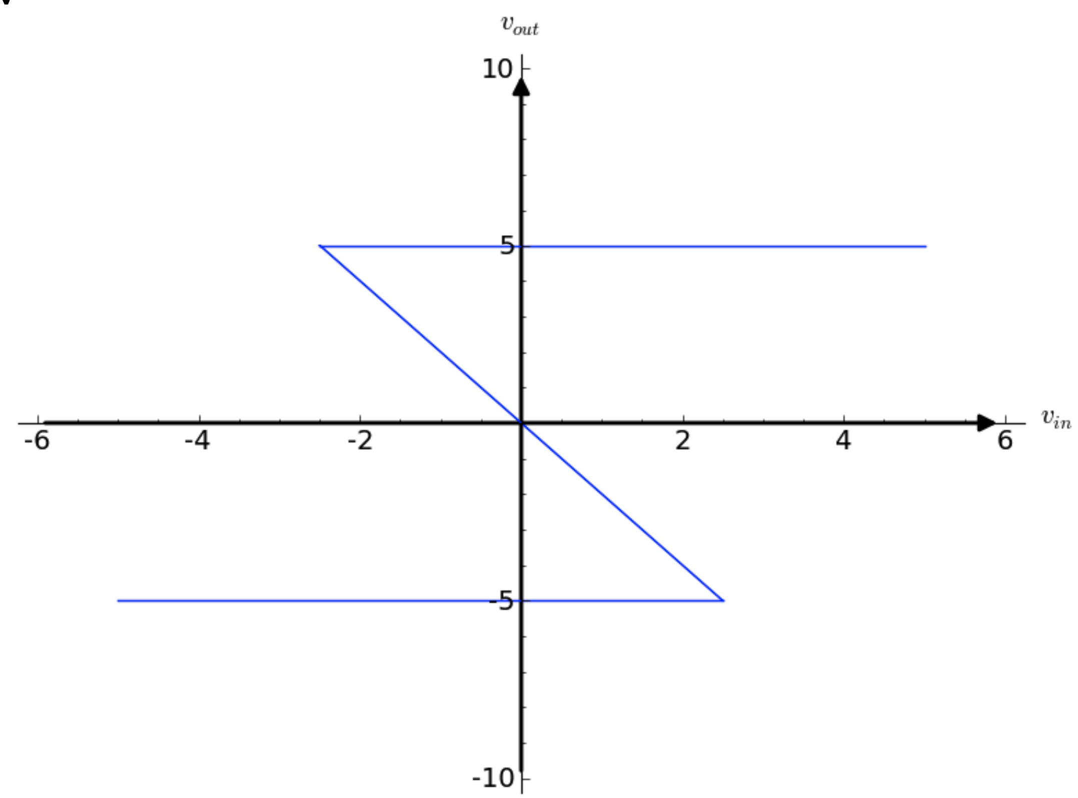
$$\Rightarrow \quad v_p = \frac{v_{in} - \frac{R_i}{R_f} v_{CC}}{1 + \frac{R_i}{R_f}}$$

Now, $v_{out} = -v_{CC}$ if $v_p \leq v_n \Rightarrow v_{out} = -v_{CC}$ if $\frac{v_{in} - \frac{R_i}{R_f} v_{CC}}{1 + \frac{R_i}{R_f}} \leq 0$

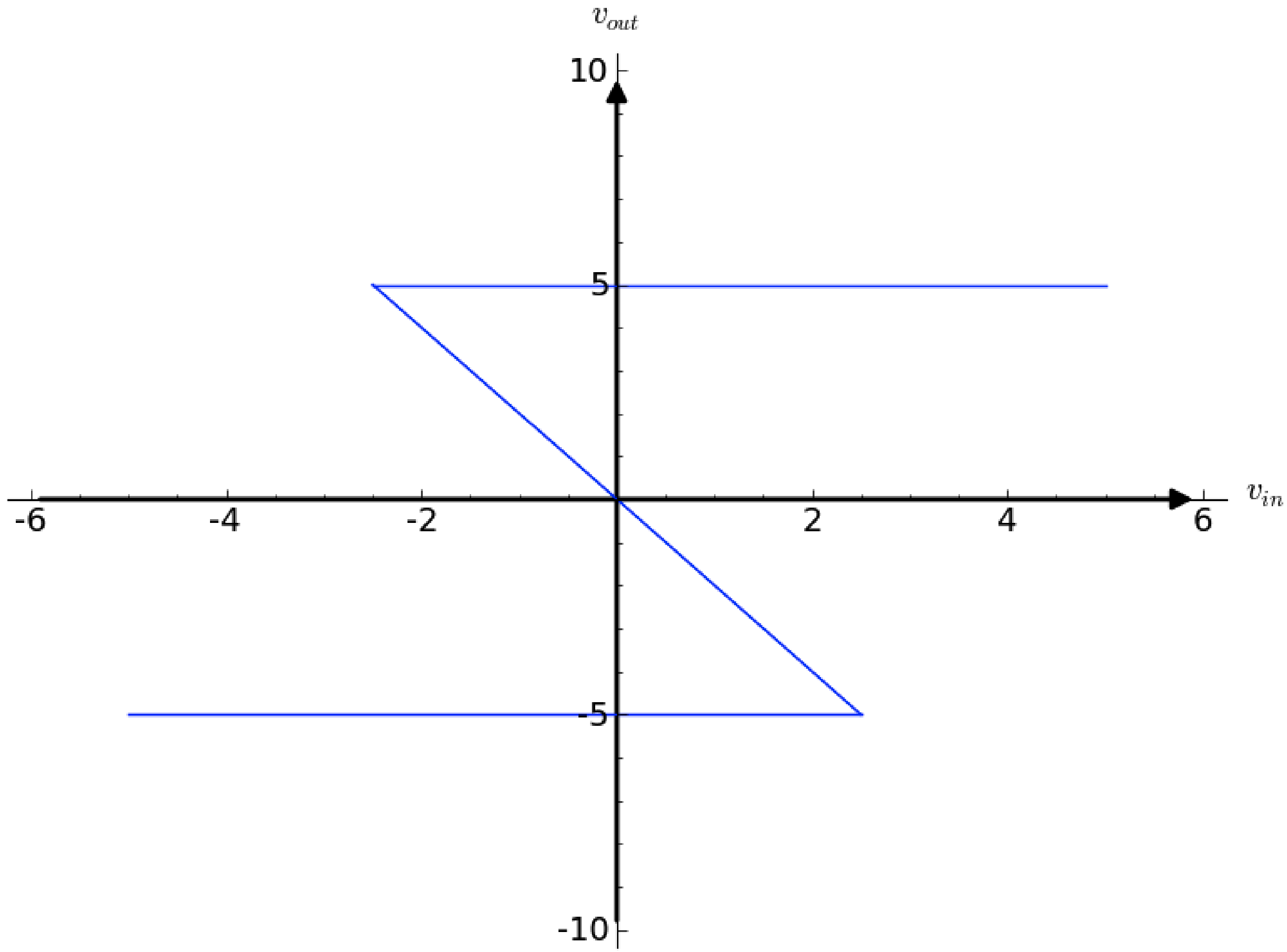
Hence: $v_{out} = -5$ if $v_{in} \leq 2.5$ V

# Step 3 (d): Sketch $V_{out}$ vs $V_{in}$

$$v_{out} = \begin{cases} 5\text{ V if } v_{in} \geq -2.5\text{ V} \\ -2v_{in} \text{ if } -2.5 < v_{in} < 2.5 \\ -5\text{ V if } v_{in} \leq 2.5\text{ V} \end{cases}$$

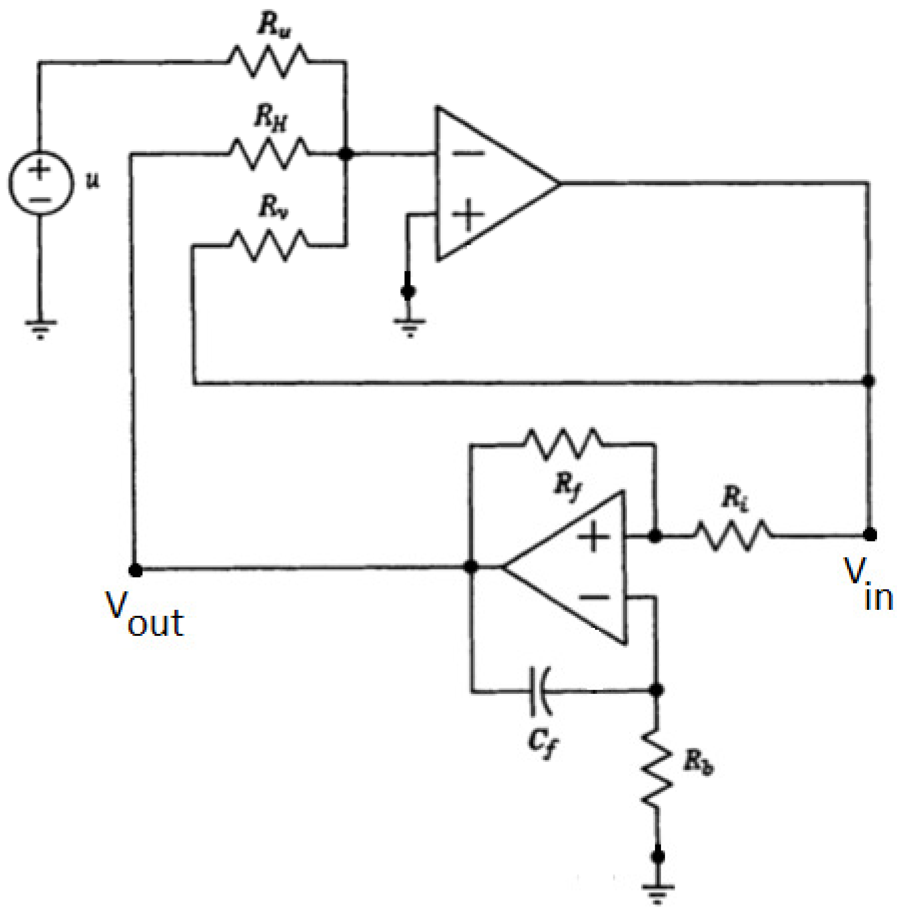


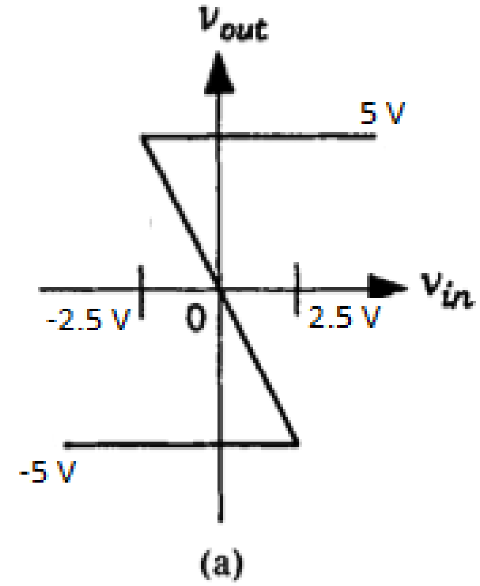
# Step 3 : Carry out the plan – Interpret the results



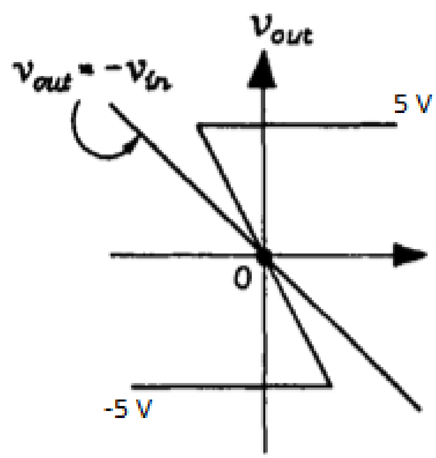
# Step 4 : Check your answer

Test Circuit $u + v_{\text{out}} + v_{\text{in}} = 0$

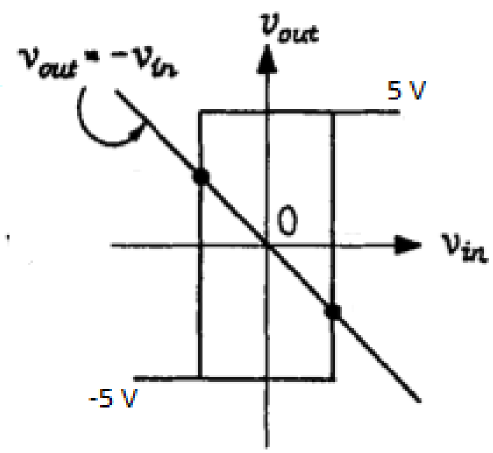



(a)

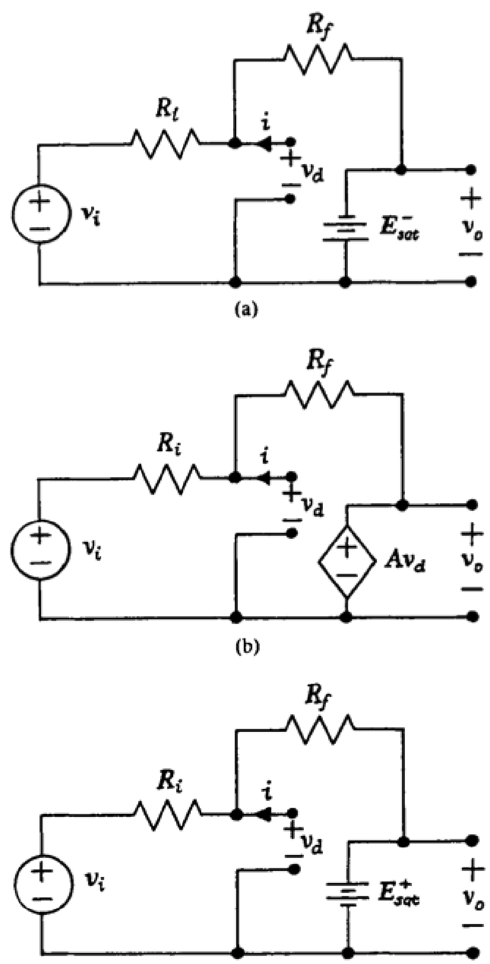



(b)

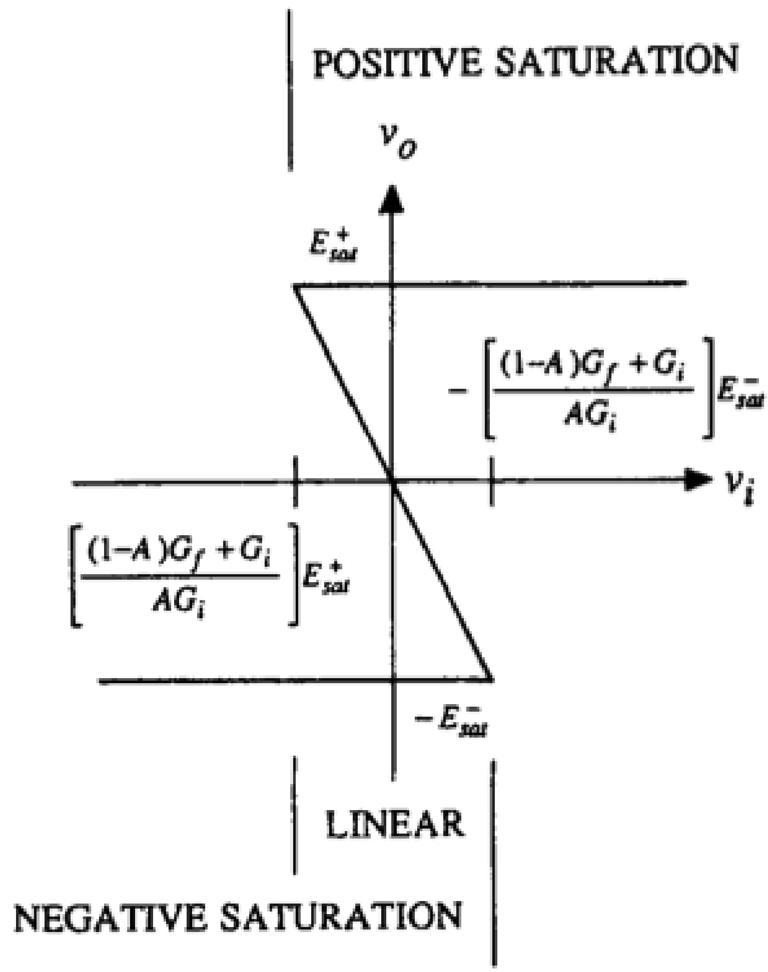



(c)

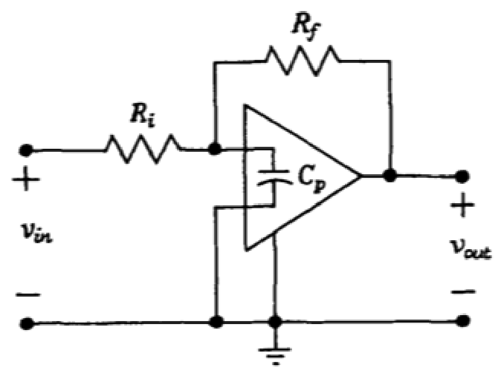
# Step 4 : Check your answer (contd.)

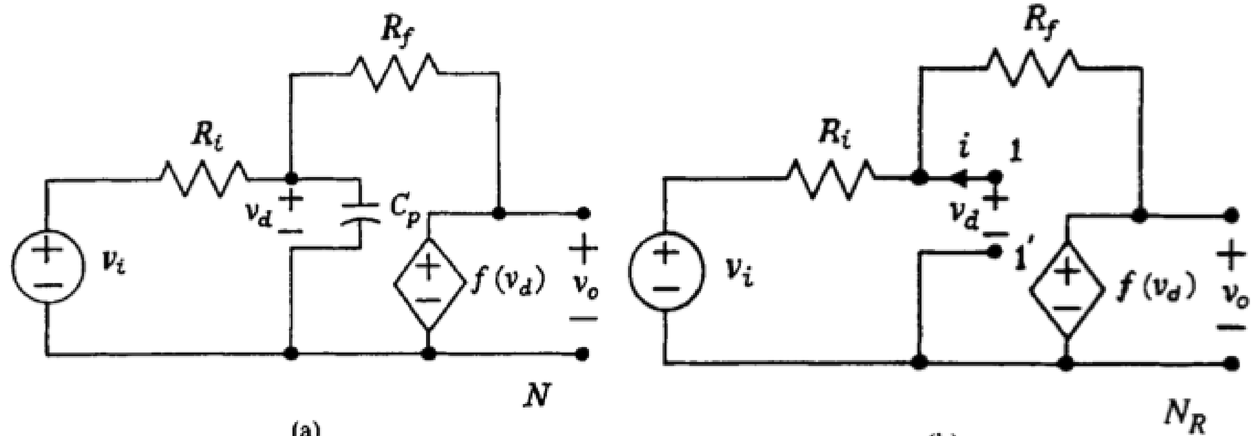


(a) (b) (c)

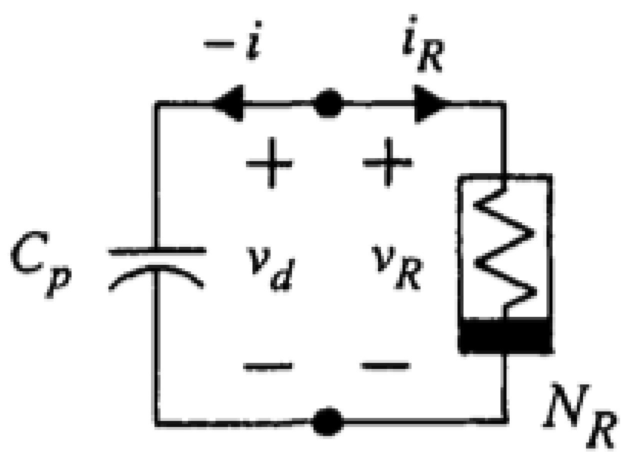


# Concept of Parasitic Capacitance

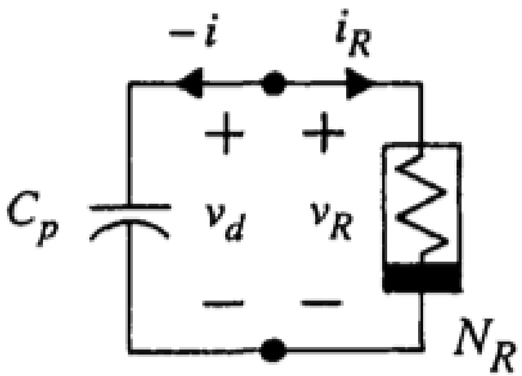


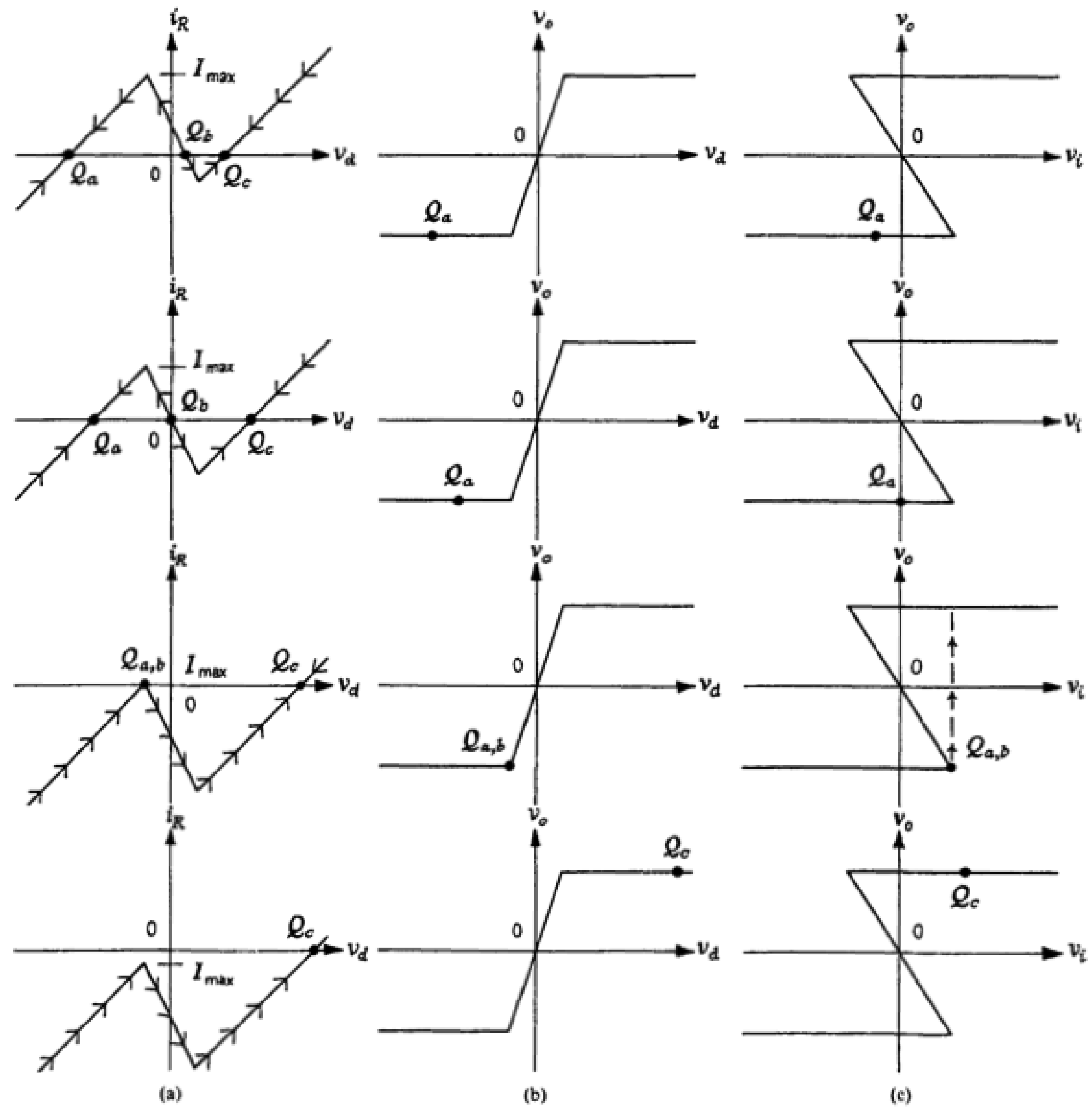




(a) (b)



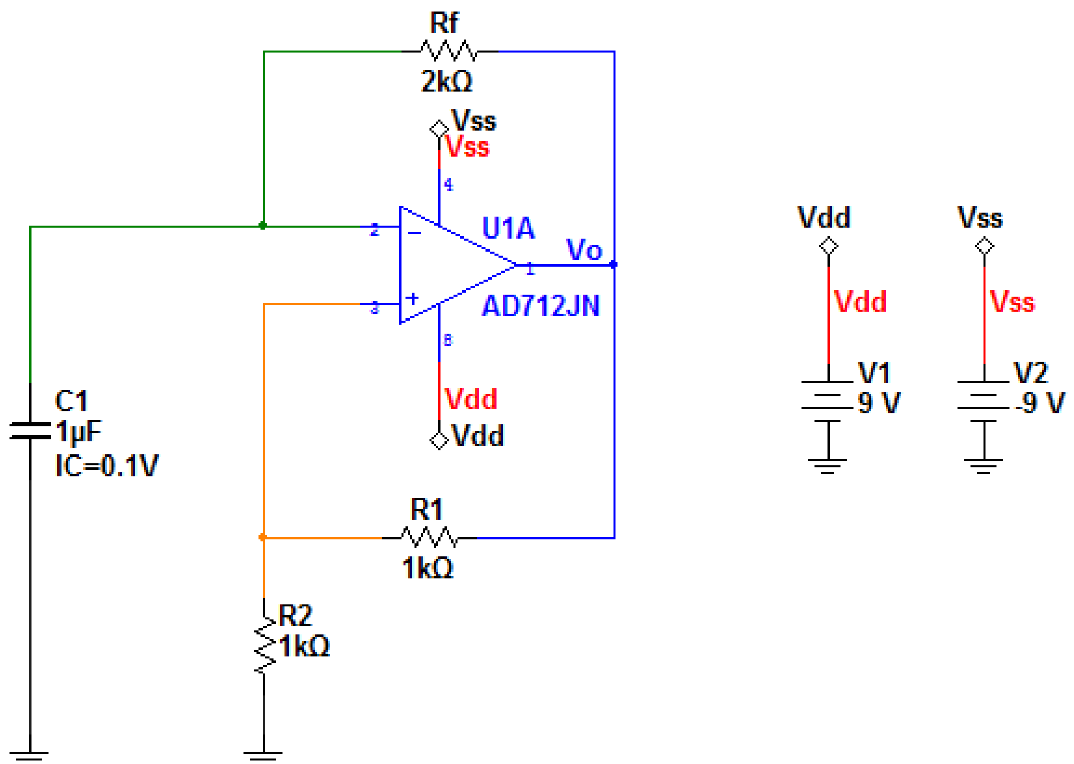
# Concept of Parasitic Capacitance (contd.)



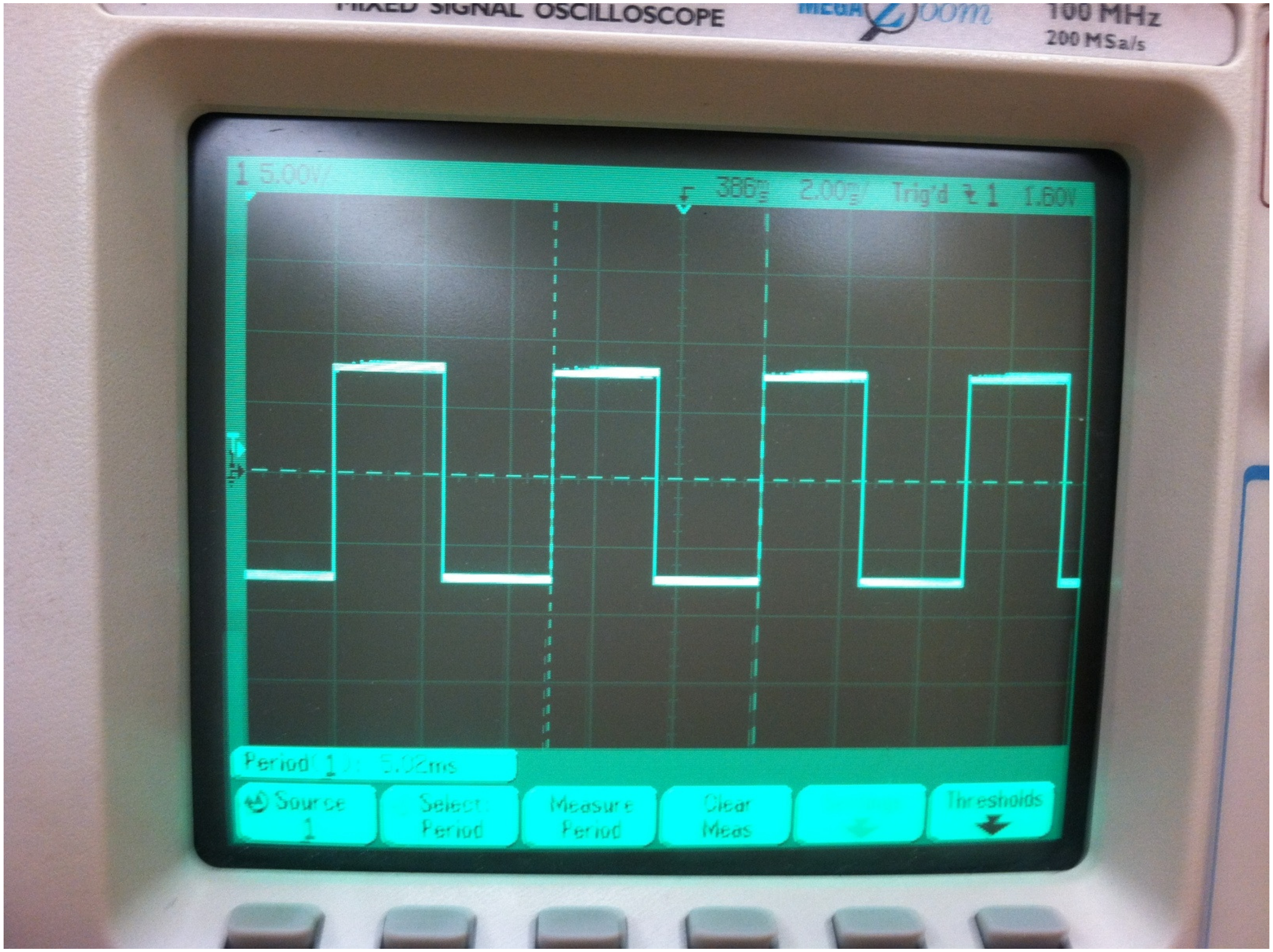



# Recall

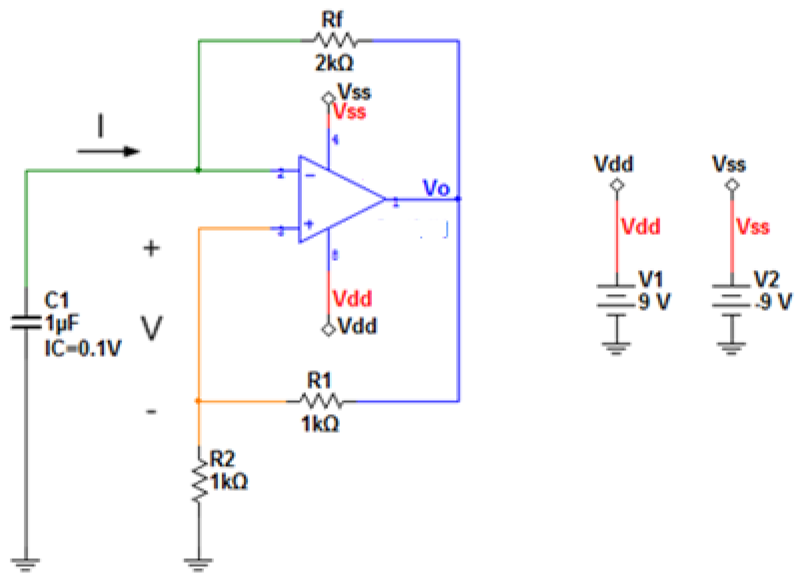
Analytically determine the period and duty cycle in the circuit below.

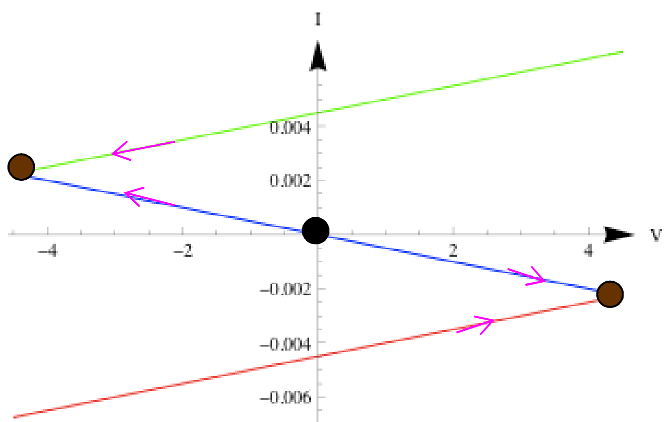


# Definition of period and duty cycle



# Finding the period




In the saturation region(s) :

$$v(t) = v_f + (v_i - v_f)e^{-t/R_f C}$$

In the positive saturation region :

$$v(t) = 9 + (-4.5 - 9)e^{-t/R_f C}$$

The period is 2x the switching time $t_S$ :

$$4.5 = 9 - 13.5e^{-\left(t_S/R_f C\right)}$$

$$\text{Period} = 2t_S = 2\ln(3)R_f C$$

# Finding the duty cycle

1. Recall (from last lecture): $v_p = \dfrac{R_2}{R_1 + R_2} v_0$

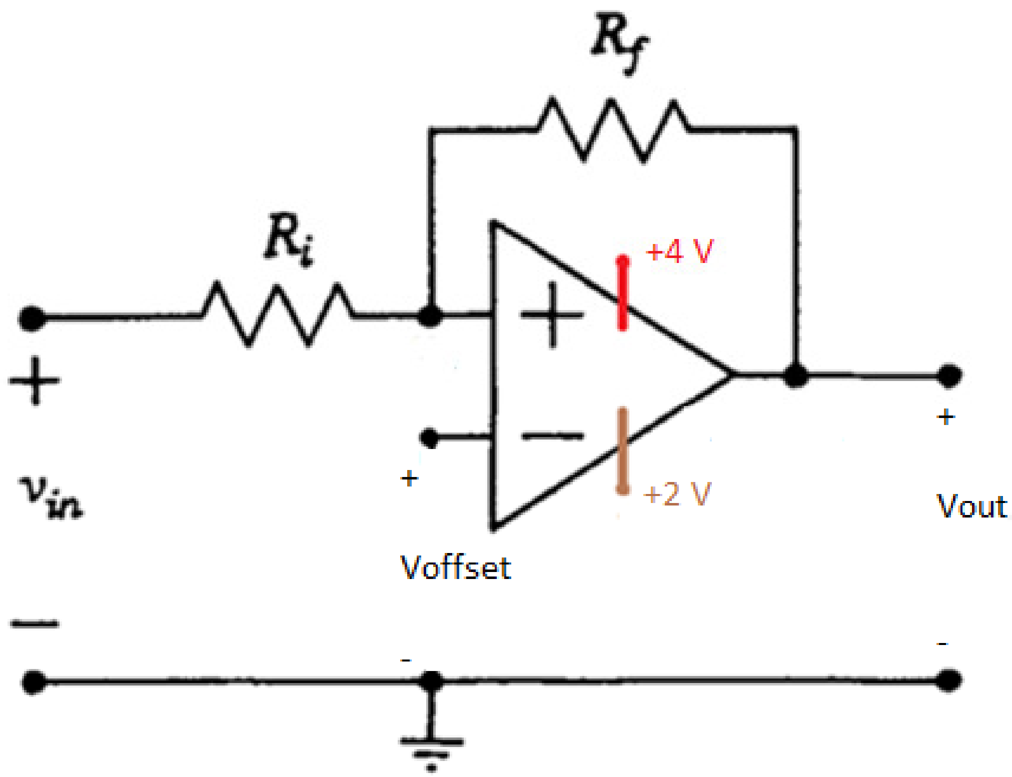
2. Circuit switches state when (positive saturation) : $v_p > v_n$

3. In other words : $V < \dfrac{R_2}{R_1 + R_2} \mathrm{V}_{dd}$

4. Hence, duty cycle is controlled by $\mathrm{V}_{dd}$ and $\mathrm{V}_{ss}$

# Conclusion

Sketch $V_{out}$ vs $V_{in}$ for the circuit below, for a constant $V_{offset}$.




Questions??